Vectors: exam practice

10 questions · 31 marks · worked solutions and markschemes

Name _____ Date _____ Score / 31

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/vectors-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

A parallelogram has sides given by the vectors $\begin{pmatrix}2\\1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\3\\1\end{pmatrix}$. Find its exact area.

2. **3 marks · Paper 1 style · calculator allowed**

Given the vectors $\vec{a} = \begin{pmatrix}2\\-1\\4\end{pmatrix}$ and $\vec{b} = \begin{pmatrix}3\\5\\k\end{pmatrix}$.

Find the value of k such that the vectors are perpendicular.

3. **2 marks · Paper 1 style · calculator allowed**

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ have $|\boldsymbol{a}| = 3$, $|\boldsymbol{b}| = 4$, and the angle between them is $30°$. Find $|\boldsymbol{a} \times \boldsymbol{b}|$.

4. 2 marks · Paper 1 style · calculator allowed

$\mathbf{a} = \begin{pmatrix} k \\ 2 \\ -1 \end{pmatrix}$ is perpendicular to $\mathbf{b} = \begin{pmatrix} 3 \\ k \\ 4 \end{pmatrix}$.

Find k. **[2]**

5. 3 marks · Paper 1 style · calculator allowed

Given that the vector product of two vectors $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b} = \begin{pmatrix} 5 \\ -5 \\ -5 \end{pmatrix}$.

Find the exact area of the triangle formed by the vectors $\vec{a}$ and $\vec{b}$.

6. 3 marks · Paper 1 style · calculator allowed

Find $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ and the area of the parallelogram formed by the two vectors.

Find the cross product and the area. **[3]**

7. **3 marks · Paper 1 style · calculator allowed**

Find the area of the triangle with vertices $A(0,0,0)$, $B(3,0,4)$ and $C(0,5,0)$.

Find the area. **[3]**

8. **3 marks · Paper 2 style · calculator allowed**

A force $\mathbf{F} = \begin{pmatrix} 2 \\ 0 \\ 5 \end{pmatrix}$ N acts at the point with position vector $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$ m.

Find the moment $\mathbf{r} \times \mathbf{F}$ and its magnitude. **[3]**

9. **5 marks · Paper 2 style · calculator allowed**

Given the vectors $\vec{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\vec{q} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$.

Calculate the angle between the two vectors. Give your answer in degrees to 3 significant figures.

10. **4 marks · Paper 2 style · calculator allowed**

Find the shortest distance from $P(2, 3, 1)$ to the line $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Find the distance. **[4]**

Answers and worked solutions

Question 1 3 marks

Answer: $\sqrt{41}$

cross product $\begin{pmatrix}1\\-2\\6\end{pmatrix}$ (M1)(A1)

area $= \sqrt{1+4+36} = \sqrt{41}$

A1

Question 2 3 marks

Answer: -0.25

If vectors are perpendicular, their scalar product is zero. M1

$\vec{a}\cdot\vec{b} = (2)(3)+(-1)(5)+(4)(k) = 0$

M1

$6-5+4k=0 \implies 1+4k=0 \implies k=-0.25$

A1

Question 3 2 marks

Answer: 6

$|a||b|\sin\theta = 3\times 4\times \frac{1}{2}$ (M1)

$= 6$ A1

Question 4 2 marks

Answer: $k = 0.8$

$3k+2k-4=0$ M1

$k = 0.8$ A1

Question 5 3 marks

Answer: $5\sqrt{3}/2$

Area of triangle $= \frac{1}{2}|\vec{a}\times\vec{b}|$. M1

$|\vec{a}\times\vec{b}| = \sqrt{5^2+(-5)^2+(-5)^2} = \sqrt{25+25+25} = \sqrt{75}$ (or $5\sqrt{3}$). A1

Area $= \frac{\sqrt{75}}{2}$ (or $\frac{5\sqrt{3}}{2}$). A1

Question 6 3 marks

Answer: 17.1, from $\begin{pmatrix}14\\-7\\7\end{pmatrix}$

$\begin{pmatrix}14\\-7\\7\end{pmatrix}$ A2

$\sqrt{294} = 17.1$ A1

Question 7 3 marks

Answer: $\frac{25}{2}$

$\overrightarrow{AB}\times\overrightarrow{AC} = \begin{pmatrix}-20\\0\\15\end{pmatrix}$ $M1A1$

$\frac{25}{2}$ A1

Question 8 3 marks

Answer: $16.9\ Nm$, from $\begin{pmatrix}15\\-5\\-6\end{pmatrix}$

$\begin{pmatrix}15\\-5\\-6\end{pmatrix}$ A2

$16.9\ Nm$ A1

Question 9 5 marks

Answer: $70.9°$

$\vec{p}\cdot\vec{q} = (1)(-2)+(2)(1)+(3)(1) = -2+2+3 = 3$

M1

$|\vec{p}| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

A1

$|\vec{q}| = \sqrt{(-2)^2+1^2+1^2} = \sqrt{6}$

A1

$\cos\theta = \frac{3}{\sqrt{14}\sqrt{6}} = \frac{3}{\sqrt{84}}$ M1

$\theta = \cos^{-1}\left(\frac{3}{\sqrt{84}}\right) \approx 70.9°$ A1

Question 10 4 marks

Answer: 1.63

$\overrightarrow{AP} = \begin{pmatrix}1\\3\\1\end{pmatrix}$ A1

$|\overrightarrow{AP}\times\mathbf{d}| = \sqrt{8}$ $M1A1$

$\frac{\sqrt{8}}{\sqrt{3}} = 1.63$ A1

Every solution is also on ibmathrevision.com/hlai/vectors-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.