IB MATHS AI HL · UNIT 3

Vectors and Kinematics: exam practice

10 questions · 39 marks · worked solutions and markschemes

Name _____ Date _____ Score / 39

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/vectors-and-kinematics-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

A boat moves with a constant velocity. At $t = 0$ hours, its position vector is $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ km. At $t = 3$ hours, its position vector is $\begin{pmatrix} 14 \\ -4 \end{pmatrix}$ km. Calculate the exact speed of the boat in km/h.

2. **3 marks · Paper 1 style · calculator allowed**

$\mathbf{p} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.

Find $2\mathbf{p} - 3\mathbf{q}$ and its magnitude. **[3]**

3. **3 marks · Paper 1 style · calculator allowed**

$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 8 \\ -1 \end{pmatrix}$. M is the midpoint of AB.

Find $\overrightarrow{AB}$ and $\overrightarrow{OM}$. **[3]**

4. **3 marks · Paper 1 style · calculator allowed**

Find the unit vector in the direction of $\begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$, and a vector of magnitude 14 in the same direction.

Find both. **[3]**

5. **4 marks · Paper 2 style · calculator allowed**

A boat starts at $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ km and moves with velocity $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ km/h.

Find its speed, its position after 2.5 hours, and when it is 20 km from the start. **[4]**

6. **4 marks · Paper 2 style · calculator allowed**

A particle has velocity $\mathbf{v}(t) = \begin{pmatrix} 2t \\ 6 - t \end{pmatrix}$ m/s and starts at the origin.

Find its position at $t = 3$ and its speed at $t = 3$. **[4]**

7. **4 marks · Paper 2 style · calculator allowed**

Ship A is at $\begin{pmatrix} 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and ship B at $\begin{pmatrix} 20 \\ 0 \end{pmatrix} + t\begin{pmatrix} -2 \\ 5 \end{pmatrix}$, distances in km and t in hours.

Find the minimum distance between the ships and when it occurs. **[4]**

8. **6 marks · Paper 1 style · calculator allowed**

A particle moves in a straight line such that its velocity at time t seconds is given by $v(t) = t^2 - 4$ m/s.

Find the exact total distance travelled by the particle from $t = 0$ to $t = 3$.

9. **2 marks · Paper 1 style · calculator allowed**

A drone flies in a straight line from $A(2, 5, 1)$ to $B(8, -3, 1)$, where distances are in km.

a. Find the displacement vector $\overrightarrow{AB}$. **[1]**

b. Find the distance flown. **[1]**

10. **7 marks · Paper 1 style · calculator allowed**

An asteroid is moving through space with position vector $\boldsymbol{r}_A = \begin{pmatrix} 50 \\ 20 \\ -10 \end{pmatrix} + t \begin{pmatrix} -8 \\ 4 \\ 2 \end{pmatrix}$. A space station is located at $S(10, 30, -5)$.

Let $D(t)$ be the distance between the asteroid and the space station at time t.

(a) Find an algebraic expression for $D(t)^2$ in terms of t. **[3 marks]**

(b) By graphing $D(t)^2$ on your GDC or using calculus, find the exact time t when the asteroid is closest to the space station, and state this minimum distance. **[4 marks]**

Answers and worked solutions

Question 1

3 marks

Answer: 5

Displacement over 3 hours:

$\begin{pmatrix}14-2\\-4-5\end{pmatrix}=\begin{pmatrix}12\\-9\end{pmatrix}$ (A1)

Divide by the time taken:

$\mathbf{v}=\frac{1}{3}\begin{pmatrix}12\\-9\end{pmatrix}=\begin{pmatrix}4\\-3\end{pmatrix}$ M1

Speed is the magnitude of the velocity:

$|\mathbf{v}|=\sqrt{4^2+(-3)^2}=\sqrt{25}=5\ \text{km/h}$ A1

Question 2

3 marks

Answer: 21.0, from $\begin{pmatrix}9\\-19\end{pmatrix}$

$\begin{pmatrix}9\\-19\end{pmatrix}$ A2

$\sqrt{442}=21.0$ A1

Question 3

3 marks

Answer: $\begin{pmatrix}6\\-6\end{pmatrix}$; $\begin{pmatrix}5\\2\end{pmatrix}$

$\begin{pmatrix}6\\-6\end{pmatrix}$ A1

$\overrightarrow{OA}+\frac{1}{2}\overrightarrow{AB}$ M1

$\begin{pmatrix}5\\2\end{pmatrix}$ A1

Question 4

3 marks

Answer: $\frac{1}{7}\begin{pmatrix}6\\-2\\3\end{pmatrix}$; $\begin{pmatrix}12\\-4\\6\end{pmatrix}$

magnitude 7 A1

$\frac{1}{7}\begin{pmatrix}6\\-2\\3\end{pmatrix}$ A1

$\begin{pmatrix}12\\-4\\6\end{pmatrix}$ A1

Question 5

4 marks

Answer: $5\ \text{km/h}$; $(9.5,\ 11)$; $t=4\ h$

$5\ \text{km/h}$ A1

$\begin{pmatrix}9.5\\11\end{pmatrix}$ A1

$5t=20$ M1

$t=4\ h$ A1

Question 6

4 marks

Answer: $(9,\ 13.5)$; $6.71\ \text{m/s}$

$\mathbf{r}=\begin{pmatrix}t^2\\6t-\frac{t^2}{2}\end{pmatrix}$ $M1$A1

$\begin{pmatrix}9\\13.5\end{pmatrix}$ A1

speed $\sqrt{36+9}=6.71\ \text{m/s}$ A1

Question 7

4 marks

Answer: 6.32 km at $t=3\ h$

$\overrightarrow{AB}=\begin{pmatrix}20-6t\\2t\end{pmatrix}$ A1

$d^2=(20-6t)^2+4t^2$ M1

$t=3\ h$ A1

$d=6.32$ km A1

Question 8

6 marks

Answer: $23/3$

Total distance is the integral of speed:

$\int_0^3|t^2-4|\,dt$ M1

Find where the velocity changes sign:

$t^2-4=0\implies t=2$ (since $t\geq 0$) M1

Split the integral at $t=2$:

$\int_0^2(4-t^2)\,dt+\int_2^3(t^2-4)\,dt$ M1

Integrate: $=\left[4t-\frac{t^3}{3}\right]_0^2+\left[\frac{t^3}{3}-4t\right]_2^3$ A1

Evaluate the limits:

$=(8-\frac{8}{3})+((9-12)-(\frac{8}{3}-8))=\frac{16}{3}+\frac{7}{3}$ M1

Total distance $=\frac{23}{3}\ m$ A1

Question 9

2 marks

Answer: (a) $(6,-8,0)$; (b) (km) 10

a. $\begin{pmatrix}6\\-8\\0\end{pmatrix}$ A1

b. $\sqrt{36+64}=10$ km A1

Question 10

7 marks

Answer: (a) $D(t)^2$) $84t^2-740t+1725$; (b) t) $185/42$; (b) minimum distance) $20\sqrt{105}/21$

(a) Vector from S to the asteroid:

$\vec{SA}=r_A-\vec{OS}$ M1

$\vec{SA}=\begin{pmatrix}50-8t-10\\20+4t-30\\-10+2t+5\end{pmatrix}=\begin{pmatrix}40-8t\\4t-10\\2t-5\end{pmatrix}$ A1

$D(t)^2=(40-8t)^2+(4t-10)^2+(2t-5)^2$ A1

(b) Expand:

$D^2=(64t^2-640t+1600)+(16t^2-80t+100)+(4t^2-20t+25)=8$

Set the derivative to zero (or find the minimum of the graph on the GDC):

$\frac{d(D^2)}{dt}=168t-740=0$ M1

$t=\frac{740}{168}=\frac{185}{42}\approx 4.40$ A1

Substitute $t=\frac{185}{42}$:

$D^2=1725-\frac{740^2}{4(84)}=\frac{2000}{21}\approx 95.2$ (M1)

Minimum distance

$D=\sqrt{\frac{2000}{21}}=\frac{20\sqrt{105}}{21}\approx 9.76$ A1

Every solution is also on ibmathrevision.com/hlai/vectors-and-kinematics-exam-practice

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