The Second Derivative and Concavity: exam practice

10 questions · 41 marks · worked solutions and markschemes

Name _____ Date _____ Score / 41

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/the-second-derivative-and-concavity-exam-practice

1. **4 marks · Paper 1 style · calculator allowed**

Given the function $f(x) = x^4 - 3x^3 + 2x^2 - 5x + 7$, find an expression for the second derivative, $f''(x)$.

2. **4 marks · Paper 1 style · calculator allowed**

Consider the function $y = e^{2x} + \sin x$.

Find the exact value of $\frac{d^2y}{dx^2}$ at the point where $x = \frac{\pi}{2}$.

3. **2 marks · Paper 1 style · calculator allowed**

The graph of $y = f'(x)$ crosses the x-axis at $x = 1$ and has a minimum at $x = 3$.

State the x-coordinates of a stationary point and a point of inflexion of $y = f(x)$. **[2]**

4. **3 marks · Paper 1 style · calculator allowed**

Let $f(x) = x^4 - 6x^2$.

Find the intervals on which f is concave down. **[3]**

5. **3 marks · Paper 2 style · calculator allowed**

The number of downloads of an app is modelled by $D(t) = \dfrac{5000}{1 + 20e^{-0.5t}}$.

Find the time when the rate of downloads is greatest. **[3]**

6. **3 marks · Paper 1 style · calculator allowed**

Let $f(x) = xe^x$.

Find the point of inflexion. **[3]**

7. **3 marks · Paper 2 style · calculator allowed**

Let $f(x) = \dfrac{1}{1 + x^2}$.

Find the x-coordinates of the points of inflexion. **[3]**

8. 5 marks · Paper 1 style · calculator allowed

Let $f(x) = x^3 - 9x^2 + 24x$.

Find the stationary points, classify them with $f''(x)$, and find the point of inflexion. **[5]**

9. 6 marks · Paper 1 style · calculator allowed

A particle moves along a straight line such that its displacement s (in metres) from a fixed origin O at time t seconds is given by $s(t) = t^3 - 9t^2 + 15t$.

Find the exact time t when the acceleration of the particle is exactly zero.

10. 8 marks · Paper 1 style · calculator allowed

A cubic function is defined by $f(x) = ax^3 + bx^2 + x$, where a and b are constants.

The curve has a local minimum at $x = 1$, and a point of inflexion at $x = \frac{2}{3}$.

Find the exact values of a and b.

Answers and worked solutions

Question 1 — 4 marks

Answer: $12x^2 - 18x + 4$

Differentiate term by term:

$f'(x) = 4x^3 - 9x^2 + 4x - 5$ **M1 A1**

Differentiate again:

$f''(x) = 12x^2 - 18x + 4$ **A1**

A1

Question 2 — 4 marks

Answer: $4e^{\pi} - 1$

First derivative:

$\frac{dy}{dx} = 2e^{2x} + \cos x$ **M1 A1**

Second derivative:

$\frac{d^2y}{dx^2} = 4e^{2x} - \sin x$ **A1**

Substitute $x = \frac{\pi}{2}$:

$4e^{\pi} - \sin\frac{\pi}{2} = 4e^{\pi} - 1$ **A1**

Question 3 — 2 marks

Answer: $x = 1$; $x = 3$

stationary point at $x = 1$ **A1**

inflexion at $x = 3$ **A1**

Question 4 — 3 marks

Answer: $-1 < x < 1$

$f''(x) = 12x^2 - 12$ **A1**

$f''(x) < 0$ **M1**

$-1 < x < 1$ **A1**

Question 5 — 3 marks

Answer: $t = 5.99$

point of inflexion of a logistic curve **R1**

$20e^{-0.5t} = 1$ **M1**

$t = 5.99$ **A1**

Question 6 — 3 marks

Answer: $(-2,\ -2e^{-2})$

$f''(x) = (x+2)e^x$ **A1**

$x = -2$ **A1**

$(-2,\ -2e^{-2})$ **A1**

Question 7 — 3 marks

Answer: $x = \pm 0.577$

$f''(x) = \dfrac{6x^2 - 2}{(1+x^2)^3}$ **A1**

$6x^2 = 2$ **M1**

$x = \pm 0.577$ **A1**

Question 8 — 5 marks

Answer: max $(2, 20)$, min $(4, 16)$, inflexion $(3, 18)$

$f'(x) = 3(x-2)(x-4)$ **A1**

$f''(2) = -6 < 0$: maximum $(2, 20)$ **A1**

$f''(4) = 6 > 0$: minimum $(4, 16)$ **A1**

$f''(x) = 6x - 18 = 0$ **M1**

inflexion $(3, 18)$ **A1**

Question 9 — 6 marks

Answer: 3

Velocity is the derivative of displacement:

$v(t) = s'(t) = 3t^2 - 18t + 15$ **M1 A1**

Acceleration is the derivative of velocity:

$a(t) = s''(t) = 6t - 18$ **M1 A1**

Set acceleration to zero:

$6t - 18 = 0$ **M1**

$t = 3$ seconds **A1**

Question 10 — 8 marks

Answer: (a) 1; (b) -2

First derivative:

$f'(x) = 3ax^2 + 2bx + 1$ **A1**

Local minimum at $x = 1$, so

$f'(1) = 0$ **M1**

$3a + 2b + 1 = 0 \implies 3a + 2b = -1$ **A1**

Second derivative:

$f''(x) = 6ax + 2b$ **A1**

Point of inflexion at $x = \frac{2}{3}$, so

$f''\left(\frac{2}{3}\right) = 0$ **M1**

$6a\left(\frac{2}{3}\right) + 2b = 0 \implies 4a + 2b = 0$ **A1**

Subtract the equations:

$(4a + 2b) - (3a + 2b) = 0 - (-1) \implies a = 1$ **A1**

Substitute into $4a + 2b = 0$:

$2b = -4 \implies b = -2$ **A1**

Check:

$f''(1) = 6(1)(1) + 2(-2) = 2 > 0$, confirming a minimum at $x = 1$

Every solution is also on ibmathrevision.com/hlai/the-second-derivative-and-concavity-exam-practice
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