IB MATHS AI HL · UNIT 5

Tangents & Normals: exam practice

10 questions · 32 marks · worked solutions and markschemes

Name _____ Date _____ Score / 32

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/tangents-and-normals-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

Find the equation of the tangent to $y = x^3 - 2x + 1$ at $x = 2$.

Find the equation. **[3]**

2. **3 marks · Paper 1 style · calculator allowed**

Find the equation of the normal to $y = \frac{4}{x}$ at $(2, 2)$.

Find the equation. **[3]**

3. **3 marks · Paper 2 style · calculator allowed**

The tangent to $y = x^2 + kx$ at $x = 1$ is parallel to $y = 5x - 2$.

Find k and the equation of the tangent. **[3]**

4. **3 marks · Paper 2 style · calculator allowed**

Find the equation of the tangent to $y = e^{2x}$ at $x = 0$, and the point where it meets the x-axis.

Find the tangent and the intercept. **[3]**

5. **3 marks · Paper 1 style · calculator allowed**

Find the coordinates of the points on $y = x^3 - 3x^2 + 2$ where the tangent is horizontal.

Find the points. **[3]**

6. **3 marks · Paper 2 style · calculator allowed**

Find the equation of the normal to $y = \sqrt{x}$ at $x = 4$.

Find the equation. **[3]**

7. **3 marks · Paper 2 style · calculator allowed**

Let $f(x) = 2x^3 - 3x^2 - 12x + 4$.

Find the equations of the tangents with gradient 0. **[3]**

8. 3 marks · Paper 2 style · calculator allowed

Find the equation of the tangent to $y = \ln x$ at $x = e$.

Find the equation. **[3]**

9. 4 marks · Paper 2 style · calculator allowed

The tangent and normal to $y = x^2$ at $(1, 1)$ meet the x-axis at A and B.

Find the length AB. **[4]**

10. 4 marks · Paper 2 style · calculator allowed

The tangent to $y = x^2$ at (a, a^2) passes through $(1, -3)$.

Find the possible values of a. **[4]**

Answers and worked solutions

Question 1 — 3 marks

Answer: $y = 10x - 15$

$\frac{dy}{dx} = 3x^2 - 2 = 10$ **A1**

point $(2, 5)$ **A1**

$y = 10x - 15$ **A1**

Question 2 — 3 marks

Answer: $y = x$

$\frac{dy}{dx} = -\frac{4}{x^2} = -1$ **A1**

normal gradient 1 **A1**

$y = x$ **A1**

Question 3 — 3 marks

Answer: $k = 3$; $y = 5x - 1$

$2 + k = 5$ **M1**

$k = 3$ **A1**

$y = 5x - 1$ **A1**

Question 4 — 3 marks

Answer: $y = 2x + 1$; $\left(-\frac{1}{2}, 0\right)$

gradient 2, point $(0, 1)$ **A1**

$y = 2x + 1$ **A1**

$\left(-\frac{1}{2},\ 0\right)$ **A1**

Question 5 — 3 marks

Answer: $(0, 2)$ and $(2, -2)$

$3x^2 - 6x = 0$ **M1**

$x = 0$, $x = 2$ **A1**

$(0, 2)$ and $(2, -2)$ **A1**

Question 6 — 3 marks

Answer: $y = -4x + 18$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{4}$ **A1**

normal gradient -4 **A1**

$y = -4x + 18$ **A1**

Question 7 — 3 marks

Answer: $y = -16$ and $y = 11$

$6x^2 - 6x - 12 = 0$ **M1**

$x = 2$, $x = -1$ **A1**

$y = -16$ and $y = 11$ **A1**

Question 8 — 3 marks

Answer: $y = \frac{x}{e}$

gradient $\frac{1}{e}$ **A1**

point $(e,\ 1)$ **A1**

$y = \frac{x}{e}$ **A1**

Question 9 — 4 marks

Answer: $\frac{5}{2}$

tangent $y = 2x - 1$: $A\left(\frac{1}{2}, 0\right)$ **A1**

normal $y = -\frac{1}{2}x + \frac{3}{2}$: $B(3, 0)$ **A1**

$3 - \frac{1}{2}$ **M1**

$\frac{5}{2}$ **A1**

Question 10 — 4 marks

Answer: $a = 3$, $a = -1$

tangent $y = 2ax - a^2$ **A1**

$-3 = 2a - a^2$ **M1**

$a^2 - 2a - 3 = 0$ **A1**

$a = 3$, $a = -1$ **A1**

Every solution is also on ibmathrevision.com/hlai/tangents-and-normals-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.