Statistical Testing: exam practice

10 questions · 35 marks · worked solutions and markschemes

Name _____ Date _____ Score / 35

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/statistical-testing-exam-practice

1. **1 mark · Paper 1 style · calculator allowed**

A contingency table has 3 rows and 4 columns.

Find the number of degrees of freedom for a test of independence. **[1]**

2. **2 marks · Paper 2 style · calculator allowed**

The number of goals per match in 100 matches is recorded and tested for a Poisson fit with mean 1.5. The χ^2 statistic is 3.82 with 3 degrees of freedom.

Find the p-value and state the conclusion at the 5% level. **[2]**

3. **2 marks · Paper 2 style · calculator allowed**

In a test of independence, one expected frequency is 3.2.

Explain why this is a problem and how it could be dealt with. **[2]**

4. **2 marks · Paper 2 style · calculator allowed**

A goodness-of-fit test has 5 categories and one parameter estimated from the data.

Find the number of degrees of freedom. **[2]**

5. **4 marks · Paper 2 style · calculator allowed**

The 2×2 table shows whether 120 patients improved after a new drug or a placebo. Test at the 1% level.

	Improved	Not improved
Drug	42	18
Placebo	28	32

Carry out the test. **[4]**

6. **5 marks · Paper 2 style · calculator allowed**

A survey of 150 people records whether they prefer tea or coffee, by age group. Test at the 5% level whether preference is independent of age.

	Under 30	30 to 50	Over 50
Tea	18	22	30
Coffee	32	28	20

Carry out the test. **[5]**

7. **4 marks · Paper 2 style · calculator allowed**

A population is normally distributed with a known standard deviation of $\sigma = 12$. A researcher wishes to estimate the population mean, μ, using a 95% confidence interval.

Determine the minimum sample size, n, required to ensure that the margin of error is no more than 3.

8. **5 marks · Paper 2 style · calculator allowed**

A spinner is supposed to land on A, B, C, D with probabilities $0.4,\ 0.3,\ 0.2,\ 0.1$. In 200 spins the results are $72,\ 66,\ 40,\ 22$. Test at the 5% level.

Carry out the test. **[5]**

9. **4 marks · Paper 2 style · calculator allowed**

A random sample of 50 students from a large university was surveyed about their weekly study hours. The sample mean was 18.5 hours with a sample standard deviation of 4.2 hours.

Calculate a 95% confidence interval for the true mean study hours of all students at the university.

10. **6 marks · Paper 2 style · calculator allowed**

A manufacturer claims that the mean breaking strength of their cables is $50\ \text{kg}$. A skeptic believes it is greater, setting up hypotheses $H_0 : \mu = 50$ and $H_1 : \mu > 50$. The breaking strength is known to be normally distributed with a standard deviation of $8\ \text{kg}$. A random sample of 30 cables is tested. The skeptic decides to reject H_0 if the sample mean, $\bar{x}$, is greater than $53\ \text{kg}$.

1. State the distribution of the sample mean, $\bar{X}$, assuming H_0 is true.
2. Calculate the probability of making a Type I error with this test.

Answers and worked solutions

Question 1 — 1 mark

Answer: 6

$(3-1)(4-1)=6$ **A1**

Question 2 — 2 marks

Answer: $p=0.282$; the Poisson model fits

$p=0.282$ **A1**

$p>0.05$: Poisson model fits **R1**

Question 3 — 2 marks

Answer: a proof: the worked solution above is the full answer

expected frequencies should be at least 5 **R1**

combine rows or columns **A1**

Question 4 — 2 marks

Answer: 3

$5-1-1$ **M1**

3 **A1**

Question 5 — 4 marks

Answer: $\chi^2=6.72$, $p=0.00953$; reject H_0

H_0: improvement independent of treatment **A1**

$\chi^2=6.72$, $\mathrm{df}=1$ **A1**

$p=0.00953$ **A1**

$p<0.01$: reject H_0 **R1**

Question 6 — 5 marks

Answer: $\chi^2=6.00$, $p=0.0498$; reject H_0

H_0: preference is independent of age **A1**

$\mathrm{df}=2$ **A1**

$\chi^2=6.00$ **A1**

$p=0.0498$ **A1**

$p<0.05$: reject H_0 **R1**

Question 7 — 4 marks

Answer: 62

For a 95% confidence interval, the z-critical value is approximately 1.96. **M1**

Margin of error formula: $E=z\times\frac{\sigma}{\sqrt{n}} \implies 3=1.96\times\frac{12}{\sqrt{n}}$. **M1**

$\sqrt{n}=\frac{1.96\times12}{3}=1.96\times4=7.84$. **A1**

$n=7.84^2\approx61.46$.

Since n must be an integer, the minimum sample size is 62. **A1**

Question 8 — 5 marks

Answer: $\chi^2=1.6$, $p=0.659$; do not reject

expected $80,\ 60,\ 40,\ 20$ **A1**

$\mathrm{df}=3$ **A1**

$\chi^2=1.6$ **A1**

$p=0.659$ **A1**

do not reject H_0 **R1**

Question 9 — 4 marks

Answer: (Lower limit) 17.3; (Upper limit) 19.7

Since population standard deviation is unknown, we use a t−distribution. **M1**

Using GDC to compute T-Interval (Stats): $\bar{x}=18.5$, $s=4.2$, $n=50$, C-Level $=0.95$. **M1**

Lower bound ≈17.3 **A1**

Upper bound ≈19.7 **A1**

Interval is $(17.3, 19.7)$.

Question 10 — 6 marks

Answer: (a) mean of $\bar{X}$ (kg) 50; (a) variance of $\bar{X}(\mathrm{kg}^2)$ $32/15$; (b) 0.0200

1. The population is normal, so the sample mean of $n=30$ cables is exactly normal with variance $\frac{\sigma^2}{n}$: $\bar{X}\sim N\left(50,\frac{8^2}{30}\right)$, so the mean is 50 and the variance is $\frac{32}{15}\approx2.13$. **A1A1**
2. Type I error occurs when H_0 is rejected while true. $P(\text{Type I Error})=P(\bar{X}>53|\mu=50)$. **M1**

   Using Normal CDF on GDC with lower $=53$, upper $=\infty$, $\mu=50$, $\sigma=\frac{8}{\sqrt{30}}\approx1.4606$. $M1$ **A1**

   Probability $=0.019990\ldots\approx0.0200$ (3 s.f.) (or 2.00%). **A1**

Every solution is also on ibmathrevision.com/hlai/statistical-testing-exam-practice
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