IB MATHS AI HL · UNIT 2

Sinusoidal Modelling: exam practice

10 questions · 32 marks · worked solutions and markschemes

Name _____ Date _____ Score / 32

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/sinusoidal-modelling-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

Let $f(x) = 2\cos(3x) - 1$.

Write down the amplitude, period and range of f. **[3]**

2. **2 marks · Paper 2 style · calculator allowed**

The temperature in a city is $T(m) = 11\sin\left(\frac{\pi}{6}(m-4)\right) + 17$, where m is the month ($m = 1$ is January).

Find the maximum temperature and the month it occurs. **[2]**

3. **2 marks · Paper 2 style · calculator allowed**

The height of a piston is $h(t) = 0.3\cos(5\pi t) + 0.5$ m, t in seconds.

Find the number of complete cycles per second and the minimum height. **[2]**

4. **3 marks · Paper 2 style · calculator allowed**

The depth of water at a dock varies between 2 m and 10 m. High tide is at $t = 0$ and the next high tide is 12.4 hours later. The depth is modelled by $d = a\cos(bt) + c$.

Find a, b and c. **[3]**

5. **3 marks · Paper 1 style · calculator allowed**

A sinusoidal model has maximum $(3,\ 14)$ and the next minimum at $(9,\ 2)$.

Find the amplitude, the principal axis and the period. **[3]**

6. **3 marks · Paper 2 style · calculator allowed**

A pendulum's horizontal displacement is $s(t) = 8\sin(\pi t)$ cm.

Find $s(0.25)$ and the times in $0 \le t \le 2$ when $s = -4$. **[3]**

7. **4 marks · Paper 2 style · calculator allowed**

The height of a seat on a Ferris wheel is $h(t) = 25 - 23\cos(12t)°$ metres, t minutes after boarding.

Find the period, the maximum height and the height after 4 minutes. **[4]**

8. **4 marks · Paper 2 style · calculator allowed**

A model $y = 3\sin(2(x-1)) + 5$ is used for $0 \le x \le 6$.

Write down the amplitude, period and range, and find the first x at which $y = 7$. **[4]**

9. **4 marks · Paper 2 style · calculator allowed**

A buoy rises and falls with the waves: $h(t) = 1.2\sin(0.8t)$ m.

Find the first time $h = 0.9$ m and the length of time in each cycle that $h > 0.9$. **[4]**

10. **4 marks · Paper 2 style · calculator allowed**

The number of daylight hours is $D(t) = 4\sin(0.0172(t-80)) + 12$, where t is the day of the year.

Find the number of days in a year with more than 15 hours of daylight. **[4]**

Answers and worked solutions

Question 1 — 3 marks

Answer: 2; $\frac{2\pi}{3}$; $-3 \leq f(x) \leq 1$

2 A1

$\frac{2\pi}{3}$ A1

$-3 \leq f(x) \leq 1$ A1

Question 2 — 2 marks

Answer: $28\,°C$ in July

$28\,°C$ A1

$m = 7$, July A1

Question 3 — 2 marks

Answer: 2.5 cycles; $0.2\ m$

period $0.4\ s$, so 2.5 cycles A1

$0.2\ m$ A1

Question 4 — 3 marks

Answer: $a = 4$, $b = 0.507$, $c = 6$

$a = 4$ A1

$c = 6$ A1

$b = \frac{2\pi}{12.4} = 0.507$ A1

Question 5 — 3 marks

Answer: 6; $y = 8$; 12

amplitude 6 A1

$y = 8$ A1

period 12 A1

Question 6 — 3 marks

Answer: 5.66 cm; $t = \frac{7}{6}, \frac{11}{6}$

$8 \sin \frac{\pi}{4} = 5.66$ A1

$\sin \pi t = -0.5$ M1

$t = \frac{7}{6}, t = \frac{11}{6}$ A1

Question 7 — 4 marks

Answer: 30 min; $48\ m$; $9.61\ m$

period 30 min A1

maximum $48\ m$ A1

$25 - 23 \cos 48°$ M1

$9.61\ m$ A1

Question 8 — 4 marks

Answer: 3, π, $2 \leq y \leq 8$; $x = 1.36$

amplitude 3, period π A1

$2 \leq y \leq 8$ A1

$\sin(2(x-1)) = \frac{2}{3}$ M1

$x = 1.36$ A1

Question 9 — 4 marks

Answer: $1.06\ s$; $1.81\ s$

$\sin 0.8t = 0.75$ M1

$t = 1.06\ s$ A1

second time $t = 2.87$ A1

$1.81\ s$ A1

Question 10 — 4 marks

Answer: 84

$\sin(0.0172(t-80)) > 0.75$ M1

$t - 80$ between 49.3 and 133.3 A1A1

84 days A1

Every solution is also on ibmathrevision.com/hlai/sinusoidal-modelling-exam-practice
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