# Scaling and Linearising Data: exam practice

10 questions · 40 marks · worked solutions and markschemes

Name _____ Date _____ Score / 40

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/scaling-and-linearising-data-exam-practice

**1.** **3 marks · Paper 1 style · calculator allowed**

The variables $t$ and $P$ are modelled by the exponential relationship $P = Ae^{kt}$. The graph of $\ln P$ against $t$ is a straight line with a gradient of $-0.15$ and a vertical intercept of $4.2$. Find the exact value of $A$.

**2.** **3 marks · Paper 2 style · calculator allowed**

A plot of $\ln y$ against $x$ is a straight line through $(0,\ 1.5)$ and $(4,\ 3.1)$.

Find the relationship between $y$ and $x$ in the form $y = ae^{bx}$. **[3]**

**3.** **3 marks · Paper 2 style · calculator allowed**

A plot of $\log_{10} y$ against $\log_{10} x$ is a straight line with gradient $3$ passing through $(0,\ 0.477)$.

Find $y$ in terms of $x$. **[3]**

**4.** 3 marks · Paper 1 style · calculator allowed

Explain how to linearise the model $y = ab^x$, and state what the gradient and intercept of the straight line give.

Explain. **[3]**

**5.** 3 marks · Paper 2 style · calculator allowed

A line of best fit for $\ln P$ against $t$ is $\ln P = -0.23t + 6.2$.

Write $P$ as a function of $t$ and find $P$ when $t = 5$. **[3]**

**6.** 3 marks · Paper 2 style · calculator allowed

The data follow $y = ax^n$.

| $x$ | 2 | 4 | 8 | 16 |
|---|---|---|---|---|
| $y$ | 12 | 34 | 96 | 272 |

Use a regression of $\ln y$ on $\ln x$ to find $a$ and $n$. **[3]**

**7.** **3 marks · Paper 1 style · calculator allowed**

The variables $x$ and $y$ are modelled by the exponential relationship $y = ae^{bx}$. The graph of $\ln y$ against $x$ forms a straight line with a gradient of $-0.5$ and a vertical intercept of $2$. Find the exact values of the constants $a$ and $b$.

**8.** **6 marks · Paper 1 style · calculator allowed**

A planet's period $P$ and distance $D$ follow a power law $P = aD^n$. On a log-log graph ($\ln P$ vs $\ln D$), the points $(0, 1.4)$ and $(2, 4.4)$ are recorded. Find the exact values of $a$ and $n$.

**9.** **6 marks · Paper 1 style · calculator allowed**

Two raw data points for an exponential relationship $y = ab^x$ are $(2, 20)$ and $(5, 150)$. By setting up simultaneous equations, find $a$ and $b$.

**10.** **7 marks · Paper 1 style · calculator allowed**

A biologist records bacteria growth. At $t = 1$, $N = 40$. At $t = 4$, $N = 1080$. Assume an exponential model $N = Ae^{kt}$. Solve algebraically for exact values of $A$ and $k$.

## Answers and worked solutions

### Question 1 — 3 marks

**Answer:** $e^{(4.2)}$

Take natural logs:

$\ln P = \ln(Ae^{kt}) = kt + \ln A$ **M1**

The vertical intercept of $\ln P$ against $t$ is $\ln A$, so $\ln A = 4.2$ **R1**

$A = e^{4.2}$ **A1**

### Question 2 — 3 marks

**Answer:** $y = 4.48e^{0.4x}$

$\ln y = 0.4x + 1.5$ **A1**

$a = e^{1.5} = 4.48$ **A1**

$b = 0.4$ **A1**

### Question 3 — 3 marks

**Answer:** $y = 3.00x^3$

$\log y = 3\log x + 0.477$ **A1**

$y = 10^{0.477}x^3$ **M1**

$y = 3.00x^3$ **A1**

### Question 4 — 3 marks

**Answer:** a proof: the worked solution above is the full answer

plot $\ln y$ against $x$ **A1**

gradient $\ln b$ **A1**

intercept $\ln a$ **A1**

### Question 5 — 3 marks

**Answer:** $156$, from $P = 493e^{-0.23t}$

$P = e^{6.2}e^{-0.23t} = 493e^{-0.23t}$

$M1$**A1**

$156$ **A1**

### Question 6 — 3 marks

**Answer:** $a = 4.24$, $n = 1.50$

GDC **(M1)**

$n = 1.50$ **A1**

$a = 4.24$ **A1**

### Question 7 — 3 marks

**Answer:** (a) $e^2$; (b) $-0.5$

Take natural logs:

$\ln y = \ln(ae^{bx}) = bx + \ln a$ **M1**

Gradient gives $b = -0.5$ **A1**

Intercept gives

$\ln a = 2 \implies a = e^2$ **A1**

### Question 8 — 6 marks

**Answer:** (a) $e^{(1.4)}$; $(n)\ 1.5$

Log-log graph, so take logs of the power model:

$\ln P = n\ln D + \ln a$ **M1**

The gradient equals $n$:

$n = \frac{4.4-1.4}{2-0}$ **M1**

$n = 1.5$ **A1**

The vertical intercept is $C = 1.4$, which equals $\ln a$: $\ln a = 1.4$ **M1**

Solve for $a$: $a = e^{1.4}$ **A1**

Exact values: $n = \frac{3}{2}$, $a = e^{1.4}$ $(\approx 4.06)$ **A1**

### Question 9 — 6 marks

**Answer:** (a) $5.22$; (b) $1.96$

Substitute both points into $y = ab^x$: $20 = ab^2$ and $150 = ab^5$ **M1**

Divide the second equation by the first: $\frac{ab^5}{ab^2} = \frac{150}{20}$ **M1**

$b^3 = 7.5$ **A1**

$b = \sqrt[3]{7.5} \approx 1.96$ (1.9574...) **A1**

Substitute the unrounded $b$ into $20 = ab^2$: $a = \frac{20}{b^2} = \frac{20}{7.5^{2/3}}$ **M1**

$a \approx 5.22$ **A1**

### Question 10 — 7 marks

**Answer:** (A) $40/3$; $(k)\ \ln(3)$

Substitute both data points:

$40 = Ae^k$ and $1080 = Ae^{4k}$ **M1 A1**

Divide to eliminate $A$:

$\frac{Ae^{4k}}{Ae^k} = \frac{1080}{40}$ **M1**

$e^{3k} = 27$ **A1**

Take natural logs:

$3k = \ln 27 \implies k = \frac{\ln 27}{3} = \ln 3$ **A1**

Substitute back:

$40 = Ae^{\ln 3} = 3A$ **M1**

$A = \frac{40}{3}$ **A1**

Every solution is also on ibmathrevision.com/hlai/scaling-and-linearising-data-exam-practice
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