# Scalar Products and Voronoi: exam practice

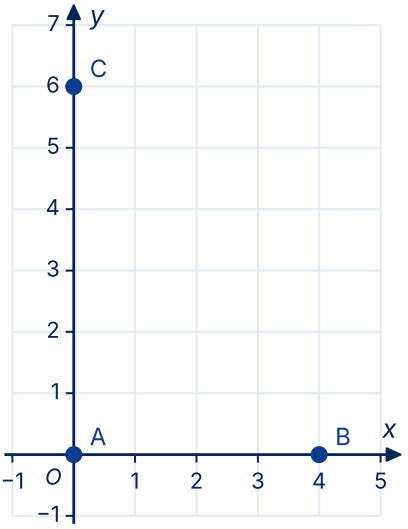
10 questions · 39 marks · worked solutions and markschemes

Name _____ Date _____ Score / 39

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/scalar-products-and-voronoi-exam-practice

**1.** **3 marks · Paper 1 style · calculator allowed**

Three sites $A(0,0)$, $B(4,0)$, and $C(0,6)$ are used to form a Voronoi diagram on a 2D plane. Find the exact coordinates of the Voronoi vertex formed between these three sites.



**2.** **4 marks · Paper 1 style · calculator allowed**

Calculate the exact scalar (dot) product of the vectors $\boldsymbol{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.

**3.** **4 marks · Paper 1 style · calculator allowed**

The vectors $\boldsymbol{p} = \begin{pmatrix} k \\ 3 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ are orthogonal (perpendicular). Find the exact value of the constant $k$.

**4.** **4 marks · Paper 1 style · calculator allowed**

A triangle has an area of $10\ \text{cm}^2$. It is transformed by the matrix $M = \begin{pmatrix} 4 & -1 \\ 2 & 3 \end{pmatrix}$.

Calculate the area of the transformed image triangle.

**5.** **2 marks · Paper 1 style · calculator allowed**

Three schools are at $A(2, 1)$, $B(8, 1)$ and $C(5, 7)$. Students go to the nearest school.

A student lives at $(6, 4)$. State which school she goes to. **[2]**

**6.** 3 marks · Paper 1 style · calculator allowed

Find the angle between $\mathbf{u} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$.

Find the angle. **[3]**

**7.** 3 marks · Paper 1 style · calculator allowed

The matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is applied to the triangle with vertices $(1, 0)$, $(3, 0)$ and $(3, 2)$.

Find the image vertices and describe the transformation. **[3]**

**8.** 5 marks · Paper 1 style · calculator allowed

Find the angle between the 3D vectors $\boldsymbol{u} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$.

Give your answer in degrees correct to 1 decimal place.

**9.** **7 marks · Paper 1 style · calculator allowed**

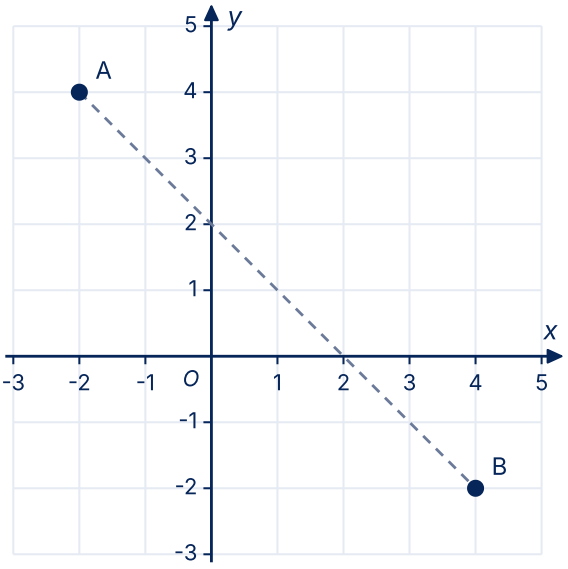
A transformation matrix is given by $A = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}$.

In matrix transformations, *invariant lines* (lines passing through the origin that map onto themselves) correspond to the **eigenvectors** of the matrix.

By solving the characteristic equation $\det(A - \lambda I) = 0$, find the two eigenvalues of matrix $A$.

**10.** **4 marks · Paper 1 style · calculator allowed**

Two sites on a Voronoi diagram are located at $A(-2, 4)$ and $B(4, -2)$. Find the coordinates of the midpoint of $[AB]$ and calculate the gradient of the perpendicular bisector separating the two sites.



## Answers and worked solutions

### Question 1 — 3 marks

**Answer:** $(2, 3)$

The Voronoi vertex is equidistant from all three sites, so it lies on the perpendicular bisectors of $AB$ and $AC$ **M1**

Bisector of $AB$ (midpoint $(2, 0)$): $x = 2$; bisector of $AC$ (midpoint $(0, 3)$): $y = 3$ **A1**

Intersection: $(2, 3)$ **A1**

### Question 2 — 4 marks

**Answer:** $2$

Scalar product:

$a \cdot b = a_1 b_1 + a_2 b_2$. **M1**

Substitute:

$a \cdot b = (3 \times 4) + (-2 \times 5)$ **A1**

$= 12 - 10$ **A1**

$= 2$ **A1**

### Question 3 — 4 marks

**Answer:** $2$

If vectors are orthogonal, their scalar (dot) product is exactly 0. **[M1]**

$p \cdot q = (k)(6) + (3)(-4) = 0$. **[M1]**

$6k - 12 = 0 \implies 6k = 12$. **[A1]**

$k = 2$. **[A1]**

### Question 4 — 4 marks

**Answer:** $140$

Area Scale Factor $= |\det(M)|$. **[M1]**

$\det(M) = (4)(3) - (-1)(2) = 12 - (-2) = 14$ . **[M1][A1]**

New Area

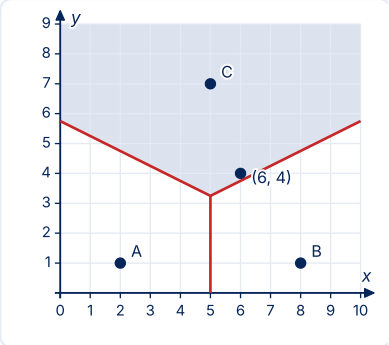
$= 10 \times 14 = 140 \text{ cm}^2$. **[A1]**

### Question 5 — 2 marks

**Answer:** school $C$

distances: $A$: $5$, $B$: $\sqrt{13}$, $C$: $\sqrt{10}$ **M1**

school $C$ **A1**




(6, 4) is in the cell of $C$

### Question 6 — 3 marks

**Answer:** $107°$

$\mathbf{u} \cdot \mathbf{v} = -5$ **A1**

$\cos\theta = \dfrac{-5}{\sqrt{14}\sqrt{21}}$ **M1**

$107°$ **A1**

### Question 7 — 3 marks

**Answer:** a proof: the worked solution above is the full answer

$(0, 1)$, $(0, 3)$, $(-2, 3)$ **A2**

rotation $90°$ anticlockwise about the origin **A1**

### Question 8 — 5 marks

**Answer:** $94.7°$

$\cos\theta = \frac{u \cdot v}{|u||v|}$. **[M1]**

$u \cdot v = (1)(3) + (2)(0) + (-1)(4) = 3 + 0 - 4 = -1$ . **[A1]**

$|u| = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6}$ .

$|v| = \sqrt{3^2 + 0^2 + 4^2} = \sqrt{25} = 5$ . **[A1]**

$\cos\theta = \frac{-1}{5\sqrt{6}}$. **[M1]**

$\theta = \cos^{-1}\left(\frac{-1}{5\sqrt{6}}\right) \approx 94.7°$. **[A1]**

### Question 9 — 7 marks

**Answer:** $6, 1$

Set $\det(A - \lambda I) = 0$. **[M1]**

$\det\begin{pmatrix} 5-\lambda & 2 \\ 2 & 2-\lambda \end{pmatrix} = 0$. **[M1]**

$(5-\lambda)(2-\lambda) - (2)(2) = 0$. **[M1]**

$10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0 \implies \lambda^2 - 7\lambda + 6 = 0$ . **[A1][A1]**

Factorise: $(\lambda - 6)(\lambda - 1) = 0$. **[M1]**

Eigenvalues are $\lambda_1 = 6$ and $\lambda_2 = 1$. **[A1]**

### Question 10 — 4 marks

**Answer:** (Midpoint) $(1, 1)$; (Gradient of the bisector) $1$

Midpoint

$= \left(\frac{-2+4}{2}, \frac{4-2}{2}\right) = (1, 1)$. **[M1]** **[A1]**

Gradient of AB

$= \frac{-2-4}{4-(-2)} = \frac{-6}{6} = -1$. **[M1]**

Perpendicular gradient

$m_\perp = \frac{-1}{m_{AB}} = +1$. **[A1]**

Every solution is also on ibmathrevision.com/hlai/scalar-products-and-voronoi-exam-practice
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