Random Variables and Poisson: exam practice

10 questions · 41 marks · worked solutions and markschemes

Name _____ Date _____ Score / 41

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/random-variables-and-poisson-exam-practice

1. **4 marks · Paper 1 style · calculator allowed**

The number of emails received by a small business in a one-hour period follows a Poisson distribution with a mean of 4.5.

Find the probability that the business receives exactly 6 emails in a randomly chosen one-hour period.

2. **2 marks · Paper 2 style · calculator allowed**

The lifetime of a component is exponential with rate $\lambda = 0.002$ per hour, so $P(T > t) = e^{-0.002t}$.

Find the mean lifetime and the probability a component lasts more than 1000 hours. **[2]**

3. **3 marks · Paper 2 style · calculator allowed**

The time between buses, T minutes, follows an exponential distribution with mean 12 minutes, so $P(T > t) = e^{-\frac{t}{12}}$.

Find the probability of waiting more than 20 minutes, and the median waiting time. **[3]**

4. **3 marks · Paper 2 style · calculator allowed**

$X \sim B(400,\ 0.01)$.

Use a Poisson approximation to estimate $P(X \leq 2)$, and give the exact binomial value. **[3]**

5. **4 marks · Paper 1 style · calculator allowed**

The number of calls received by a call center per hour follows a Poisson distribution with a mean of 4. Given that the call center receives at least 1 call in a specific hour, find the exact probability that it receives exactly 3 calls. Give your answer in terms of e.

6. **4 marks · Paper 2 style · calculator allowed**

Calls arrive at a help desk at an average of 3.5 per hour.

Find the probability of exactly 5 calls in an hour, and of more than 8 calls in two hours. **[4]**

7. **5 marks · Paper 1 style · calculator allowed**

The weights of apples are normally distributed with a mean of 150g and a standard deviation of 10g.
If two apples are chosen at random, find the probability that their combined weight is greater than 315g.

8. **3 marks · Paper 2 style · calculator allowed**

$X \sim B(100,\ 0.4)$.

Use a normal approximation with continuity correction to estimate $P(X \leq 35)$. **[3]**

9. **4 marks · Paper 1 style · calculator allowed**

A random variable X has an expected value $E(X) = 8$ and a variance $Var(X) = 5$.
A new variable is defined by the linear transformation $Y = 3X - 4$.
Calculate $E(Y)$ and $Var(Y)$.

10. **9 marks · Paper 1 style · calculator allowed**

An elevator has a strict maximum weight limit of 450 kg.

5 people enter the elevator. The weight of a person in this building is normally distributed with a mean of 80 kg and a standard deviation of 10 kg.

(a) State the distribution of the total combined weight, W_{Total}, of the 5 independent people. **[4 marks]**

(b) Calculate the probability that the elevator's weight limit is exceeded. **[5 marks]**

Answers and worked solutions

Question 1 — 4 marks

Answer: 0.128

Let $X \sim Po(4.5)$. We need $P(X = 6)$. **[M1]**

Using the formula:

$P(X = 6) = \frac{e^{-4.5} \times 4.5^6}{6!}$. **[M1][A1]**

$= 0.12812... \approx 0.128$ (3 s.f.). **[A1]**

(Or using GDC: PoissonPpd, $\lambda = 4.5, x = 6 \implies 0.128$.)

Question 2 — 2 marks

Answer: 500 hours; 0.135

500 hours **A1**

$e^{-2} = 0.135$ **A1**

Question 3 — 3 marks

Answer: 0.189; 8.32 min

$e^{-\frac{20}{12}} = 0.189$ **A1**

$e^{-\frac{m}{12}} = 0.5$ **M1**

$m = 8.32$ min **A1**

Question 4 — 3 marks

Answer: 0.238; exact 0.237

$\lambda = 4$ **A1**

$P(Y \leq 2) = 0.238$ **A1**

exact 0.237 **A1**

Question 5 — 4 marks

Answer: $32e^(-4)/(3(1-e^(-4)))$

Use conditional probability; $X = 3$ is contained in $X \geq 1$:

$P(X = 3 \mid X \geq 1) = \frac{P(X=3)}{P(X \geq 1)}$ **M1**

Numerator:

$P(X = 3) = \frac{4^3 e^{-4}}{3!} = \frac{32e^{-4}}{3}$ **A1**

Denominator:

$P(X \geq 1) = 1 - P(X = 0) = 1 - e^{-4}$ **A1**

Combine:

$P(X = 3 \mid X \geq 1) = \frac{32e^{-4}}{3(1-e^{-4})}$ **A1**

Question 6 — 4 marks

Answer: 0.132; 0.271

$P(X = 5) = 0.132$ **A1**

$\lambda = 7$ for two hours **M1**

$1 - P(Y \leq 8)$ **M1**

0.271 **A1**

Question 7 — 5 marks

Answer: 0.144

Let A be an apple.

$A_1 \sim N(150, 10^2)$ and $A_2 \sim N(150, 10^2)$.

Let $T = A_1 + A_2$.

$E(T) = 150 + 150 = 300$. **[M1]**

$Var(T) = 10^2 + 10^2 = 100 + 100 = 200$. **[M1][A1]**

Standard Deviation of $T = \sqrt{200} \approx 14.14$. **[A1]**

Find $P(T > 315)$ using Normal CDF on GDC with mean 300 and SD $\sqrt{200}$.

$P(T > 315) \approx 0.144$ (3 s.f.). **[A1]**

Question 8 — 3 marks

Answer: 0.179

$N(40,\ 24)$ **A1**

$P(Y < 35.5)$ **M1**

0.179 **A1**

Question 9 — 4 marks

Answer: $(E(Y)\ 20$; $(\mathrm{Var}(Y)\ 45$

For expected values:

$E(aX + b) = aE(X) + b$. **[M1]**

$E(Y) = 3(8) - 4 = 24 - 4 = 20$. **[A1]**

For variance (shift does not affect spread):

$Var(aX + b) = a^2 Var(X)$. **[M1]**

$Var(Y) = 3^2(5) = 9 \times 5 = 45$. **[A1]**

Question 10 — 9 marks

Answer: (a) mean (kg) 400; (a) variance) 500; (b) 0.0127

(a) Let a person's weight be $X \sim N(80, 10^2)$.

Total weight $W_T = X_1 + X_2 + X_3 + X_4 + X_5$.

$E(W_T) = 5 \times 80 = 400$ kg. $M1$ **A1**

$Var(W_T) = 5 \times 10^2 = 5 \times 100 = 500$. **M1**

Since it is the sum of normal distributions, it is also normal.

$W_T \sim N(400, 500)$. **A1**

(b) The limit is exceeded if $W_T > 450$. **M1**

Using Normal CDF on GDC: **M1**

Lower $= 450$. Upper $= 1E99$.

Mean $\mu = 400$.

Standard Deviation $\sigma = \sqrt{500} \approx 22.36$. $M1$**A1**

$P(W_T > 450) \approx 0.0127$ (or 1.27%). **A1**

Every solution is also on ibmathrevision.com/hlai/random-variables-and-poisson-exam-practice
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