Probability Distributions: exam practice

10 questions · 34 marks · worked solutions and markschemes

Name _____ Date _____ Score / 34

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/probability-distributions-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

The number of phone calls received by a call center follows a Poisson distribution with an average of 4.5 calls per minute.

Find the exact probability that exactly 6 calls are received in a given minute.

2. **3 marks · Paper 2 style · calculator allowed**

Heights of plants are $N(42,\ 6^2)$ cm.

Find the probability a plant is taller than 50 cm, and the height exceeded by 25% of plants. **[3]**

3. **3 marks · Paper 2 style · calculator allowed**

A basketball player makes 70% of her free throws. She takes 12 throws.

Find the probability that she makes at least 10, and the standard deviation of the number made. **[3]**

4. 3 marks · Paper 2 style · calculator allowed

$X \sim N(80,\ \sigma^2)$ and $P(X < 74) = 0.15$.

Find σ. **[3]**

5. 4 marks · Paper 2 style · calculator allowed

$X \sim B(15,\ 0.35)$.

Find $P(X = 5)$, $P(X \leq 3)$ and $E(X)$. **[4]**

6. 3 marks · Paper 2 style · calculator allowed

$X \sim B(n,\ 0.2)$ and $P(X \geq 1) > 0.95$.

Find the least n. **[3]**

7. 4 marks · Paper 2 style · calculator allowed

The time to complete a task is $N(\mu,\ \sigma^2)$. 10% take less than 20 minutes and 20% take more than 35 minutes.

Find μ and σ. **[4]**

8. **3 marks · Paper 1 style · calculator allowed**

A continuous random variable X has an expected value $E(X) = 20$ and a variance $\mathrm{Var}(X) = 5$.

A new variable W is defined by the linear transformation $W = 3X - 4$. Calculate the exact values of $E(W)$ and $\mathrm{Var}(W)$.

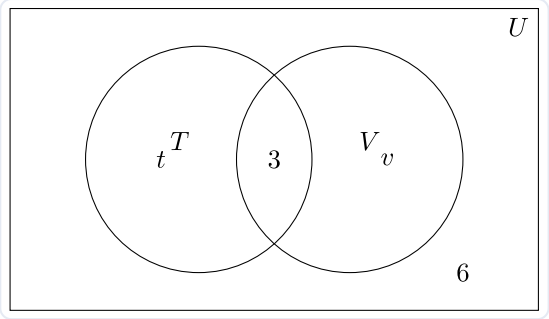
9. **4 marks · Paper 2 style · calculator allowed**

The weights of standard packages produced by a machine are normally distributed with a mean of $50\ \mathrm{kg}$ and a variance of $16\ \mathrm{kg}^2$. Three packages are selected at random.

Let $W = X_1 + X_2 + X_3$ be the total weight of the three packages. Find the mean and variance of W.

10. **4 marks · Paper 1 style · calculator allowed**

In a class of 30 students, 19 play tennis, 3 play both tennis and volleyball, and 6 do not play either sport. The following Venn diagram shows the events "plays tennis" (T) and "plays volleyball" (V).



1. Find the value of t.
2. Find the value of v.

Answers and worked solutions

Question 1 — 3 marks

Answer: 0.128

Let $X \sim Po(4.5)$. We want $P(X = 6)$. **M1**

Using Poisson PDF formula or GDC: $P(X = 6) = \frac{e^{-4.5}4.5^6}{6!}$ **M1**

≈ 0.128 **A1**

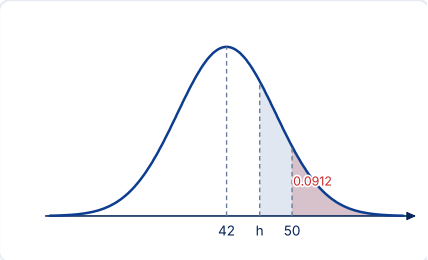
Question 2 — 3 marks

Answer: 0.0912; 46.0 cm

0.0912 **A1**

$P(H > h) = 0.25$ **M1**

46.0 cm **A1**



$P(H > 50) = 0.0912$; 25% above h

Question 3 — 3 marks

Answer: 0.253; 1.59

$1 - P(X \leq 9) = 0.253$ $M1$A1

$\sqrt{12(0.7)(0.3)} = 1.59$ **A1**

Question 4 — 3 marks

Answer: $\sigma = 5.79$

$\frac{74 - 80}{\sigma} = -1.0364$ $M1$A1

$\sigma = 5.79$ **A1**

Question 5 — 4 marks

Answer: 0.212; 0.173; 5.25

0.212 **A1**

0.173 **A1**

$15(0.35)$ **M1**

5.25 **A1**

Question 6 — 3 marks

Answer: 14

$1 - 0.8^n > 0.95$ **M1**

$n > 13.4$ **A1**

$n = 14$ **A1**

Question 7 — 4 marks

Answer: $\mu = 29.1$, $\sigma = 7.06$

$\frac{20 - \mu}{\sigma} = -1.2816$ **A1**

$\frac{35 - \mu}{\sigma} = 0.8416$ **A1**

solve **M1**

$\mu = 29.1$, $\sigma = 7.06$ **A1**

Question 8 — 3 marks

Answer: ($E(W)$ 56; ($\text{Var}(W)$ 45

Apply the expectation rule $E(aX + b) = aE(X) + b$:

$E(W) = 3E(X) - 4 = 3(20) - 4$ **M1**

$E(W) = 56$ **A1**

Apply the variance rule $\text{Var}(aX + b) = a^2\text{Var}(X)$:

$\text{Var}(W) = 3^2 \times 5 = 45$ **A1**

Question 9 — 4 marks

Answer: (Mean (kg) 150; (Variance (kg^2) 48

Mean

$E(W) = E(X_1) + E(X_2) + E(X_3)$. **M1**

$= 50 + 50 + 50 = 150$ kg. **A1**

Variance

$Var(W) = Var(X_1) + Var(X_2) + Var(X_3)$ (since they are independent). **M1**

$= 16 + 16 + 16 = 48$ kg^2. **A1**

Question 10 — 4 marks

Answer: (t) 16; (v) 5

1. The total number playing tennis is 19, so
   $t + 3 = 19$ **M1**
   $t = 16$ **A1**
2. The total number of students is 30, so
   $16 + 3 + v + 6 = 30$ **M1**
   $25 + v = 30 \implies v = 5$ **A1**

Every solution is also on ibmathrevision.com/hlai/probability-distributions-exam-practice
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