Optimisation: exam practice

10 questions · 34 marks · worked solutions and markschemes

Name _____ Date _____ Score / 34

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/optimisation-exam-practice

1. **3 marks · Paper 2 style · calculator allowed**

A cylindrical can holds $330\ \text{cm}^3$.

Find the radius that minimises the surface area of the closed can. **[3]**

2. **3 marks · Paper 2 style · calculator allowed**

A ticket price of $\$p$ gives attendance of $N = 1200 - 40p$.

Find the price that maximises revenue and the maximum revenue. **[3]**

3. **3 marks · Paper 2 style · calculator allowed**

The cost per hour of running a boat at v km/h is $C = 0.1v^2 + \dfrac{640}{v}$.

Find the speed that minimises the cost and the minimum cost. **[3]**

4. **3 marks · Paper 2 style · calculator allowed**

Two positive numbers have a sum of 30.

Find the numbers for which the product of one with the square of the other is greatest. **[3]**

5. **3 marks · Paper 2 style · calculator allowed**

A rectangle is inscribed in a semicircle of radius 10 cm, with one side on the diameter.

Find the maximum area. **[3]**

6. **3 marks · Paper 2 style · calculator allowed**

The number of bees visiting a flower bed is $B(t) = 50te^{-0.25t}$, t hours after sunrise.

Find the time of maximum visits and the maximum number. **[3]**

7. **4 marks · Paper 2 style · calculator allowed**

An open-top box with a square base has volume $4000\ \text{cm}^3$.

Find the dimensions that minimise the surface area. **[4]**

8. **4 marks · Paper 2 style · calculator allowed**

A farmer has 400 m of fencing to make two adjacent rectangular pens with a shared middle fence.

Find the maximum total area. **[4]**

9. **4 marks · Paper 2 style · calculator allowed**

A window is a rectangle topped by a semicircle. Its perimeter is 6 m. The rectangle has width $2r$.

Find r that maximises the area of the window. **[4]**

10. **4 marks · Paper 2 style · calculator allowed**

A 24 cm wire is cut into two pieces: one bent into a square, the other into a circle.

Find the length used for the square that minimises the total area. **[4]**

Answers and worked solutions

Question 1 — 3 marks

Answer: 3.74 cm

$S = 2\pi r^2 + \frac{660}{r}$ $M1$ **A1**

$r = 3.74$ cm **A1**

Question 2 — 3 marks

Answer: $\$15$; $\$9000$

$R = p(1200 - 40p)$ **M1**

$p = 15$ **A1**

$\$9000$ **A1**

Question 3 — 3 marks

Answer: $14.7\,\text{km/h}$; 65.1

$0.2v - \frac{640}{v^2} = 0$ **M1**

$v = 14.7\,\text{km/h}$ **A1**

65.1 **A1**

Question 4 — 3 marks

Answer: 20 and 10

$P = x^2(30 - x)$ **M1**

$60x - 3x^2 = 0$ **A1**

20 and 10 **A1**

Question 5 — 3 marks

Answer: $100\,\text{cm}^2$

$A = 2x\sqrt{100 - x^2}$ **M1**

$x = \sqrt{50}$ **A1**

$100\,\text{cm}^2$ **A1**

Question 6 — 3 marks

Answer: $t = 4$; 73.6

$B'(t) = 50e^{-0.25t}(1 - 0.25t)$ **M1**

$t = 4\,h$ **A1**

73.6 **A1**

Question 7 — 4 marks

Answer: base 20 cm, height 10 cm

$S = x^2 + \frac{16000}{x}$ $M1$ **A1**

$x = 20$ cm **A1**

height 10 cm **A1**

Question 8 — 4 marks

Answer: $6670\,\text{m}^2$

$3x + 2y = 400$ **A1**

$A = xy = x\left(200 - \frac{3x}{2}\right)$ **M1**

$x = \frac{200}{3}$ **A1**

$6670\,\text{m}^2$ **A1**

Question 9 — 4 marks

Answer: $0.840\,m$

height $h = \frac{6 - 2r - \pi r}{2}$ **A1**

$A = 2rh + \frac{\pi r^2}{2}$ **M1**

$\frac{dA}{dr} = 0$ **M1**

$r = 0.840\,m$ **A1**

Question 10 — 4 marks

Answer: 13.4 cm

$A = \left(\frac{x}{4}\right)^2 + \frac{(24 - x)^2}{4\pi}$ $M1$ **A1**

$\frac{x}{8} - \frac{24 - x}{2\pi} = 0$ **M1**

$x = 13.4$ cm **A1**

Every solution is also on ibmathrevision.com/hlai/optimisation-exam-practice
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