Matrices: exam practice

10 questions · 38 marks · worked solutions and markschemes

Name _____ Date _____ Score / 38

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/matrices-exam-practice

1. **4 marks · Paper 1 style · calculator allowed**

Let $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 0 \\ -2 & 5 \end{pmatrix}$.

Find $\mathbf{AB}$ and $2\mathbf{A} - \mathbf{B}$. **[4]**

2. **2 marks · Paper 1 style · calculator allowed**

The matrix $\mathbf{T} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ maps a shape of area $6\ \text{units}^2$ to its image.

Find the area of the image. **[2]**

3. 3 marks · Paper 1 style · calculator allowed

Let $\mathbf{C} = \begin{pmatrix} k & 3 \\ 2 & k-1 \end{pmatrix}$.

Find the values of k for which $\mathbf{C}$ has no inverse. **[3]**

4. 3 marks · Paper 1 style · calculator allowed

A matrix $\mathbf{A}$ has $\mathbf{A} = \mathbf{PDP}^{-1}$ with $\mathbf{D} = \begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix}$.

Write down $\mathbf{D}^{10}$ and describe what happens to $\mathbf{D}^n$ as $n \to \infty$. **[3]**

5. 3 marks · Paper 1 style · calculator allowed

Let $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.

Find $\mathbf{A}^2$ and $\mathbf{A}^3$, and write down $\mathbf{A}^n$. **[3]**

6. 4 marks · Paper 1 style · calculator allowed

Find the exact eigenvalues of the matrix $A = \begin{pmatrix} 2 & 4 \\ 1 & -1 \end{pmatrix}$.

7. **4 marks · Paper 1 style · calculator allowed**

Find the eigenvalues of the matrix $A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$.

8. **4 marks · Paper 1 style · calculator allowed**

A 2×2 matrix A has a characteristic polynomial given by $\lambda^2 - 5\lambda + 6 = 0$.

1. Find the eigenvalues of A.
2. State the exact value of the determinant of A.

9. **4 marks · Paper 1 style · calculator allowed**

Let $\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 7 & 3 \end{pmatrix}$.

(a) Find $\mathbf{A}^{-1}$. **[2]**

(b) Hence solve $5x + 2y = 4$, $7x + 3y = 5$. **[2]**

10. **7 marks · Paper 2 style · calculator allowed**

Let $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$.

(a) Write $\mathbf{A}$ in the form $\mathbf{PDP}^{-1}$, where $\mathbf{D}$ is diagonal. **[4]**

(b) Hence find $\mathbf{A}^n$. **[3]**

Answers and worked solutions

Question 1 — 4 marks

Answer: a proof: the worked solution above is the full answer

$$\mathbf{AB} = \begin{pmatrix} 4 & -5 \\ -5 & 20 \end{pmatrix}$$ A2

$$2\mathbf{A} - \mathbf{B} = \begin{pmatrix} 3 & -2 \\ 8 & 3 \end{pmatrix}$$ A2

Question 2 — 2 marks

Answer: 30 units^2

$\det \mathbf{T} = 5$ A1

30 units^2 A1

Question 3 — 3 marks

Answer: $k = 3$, $k = -2$

$\det \mathbf{C} = k(k-1) - 6 = 0$ M1

$k^2 - k - 6 = 0$ A1

$k = 3$, $k = -2$ A1

Question 4 — 3 marks

Answer: a proof: the worked solution above is the full answer

$$\begin{pmatrix} 0.5^{10} & 0 \\ 0 & 1 \end{pmatrix}$$ A1

$0.5^n \to 0$ R1

$$\mathbf{D}^n \to \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$ A1

Question 5 — 3 marks

Answer: a proof: the worked solution above is the full answer

$$\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$$ A1

$$\begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix}$$ A1

$$\begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$$ A1

Question 6 — 4 marks

Answer: $3, -2$

Characteristic equation:

$\det(A - \lambda I) = 0$ M1

$$\det \begin{pmatrix} 2-\lambda & 4 \\ 1 & -1-\lambda \end{pmatrix} = 0$$ A1

$(2-\lambda)(-1-\lambda) - 4 = 0 \implies -2 - 2\lambda + \lambda + \lambda^2 - 4 = 0$ M1

$\lambda^2 - \lambda - 6 = 0 \implies (\lambda - 3)(\lambda + 2) = 0$.

Eigenvalues are $\lambda_1 = 3$, $\lambda_2 = -2$ A1

Question 7 — 4 marks

Answer: $5, 2$

Set up the characteristic equation

$\det(A - \lambda I) = 0$ M1

$\det \begin{pmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{pmatrix} = (4-\lambda)(3-\lambda) - (1)(2) = 0$ A1

Expand: $\lambda^2 - 7\lambda + 10 = 0$ A1

Factorise:

$(\lambda - 5)(\lambda - 2) = 0 \implies \lambda = 5$ or $\lambda = 2$ A1

Question 8 — 4 marks

Answer: (a) $2, 3$; (b) 6

1. $\lambda^2 - 5\lambda + 6 = 0 \implies (\lambda - 2)(\lambda - 3) = 0$ M1
   Eigenvalues are $\lambda_1 = 2$ and $\lambda_2 = 3$ A1
2. The determinant of a matrix is the product of its eigenvalues M1
   $\det(A) = 2 \times 3 = 6$ A1

Question 9 — 4 marks

Answer: (a) $\begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}$; (b) $x = 2$, $y = -3$

(a)

$\det \mathbf{A} = 1$ A1

$$\begin{pmatrix} 3 & -2 \\ -7 & 5 \end{pmatrix}$$ A1

(b)

$\mathbf{A}^{-1} \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ M1

$x = 2$, $y = -3$ A1

Question 10 — 7 marks

Answer: (b) $\begin{pmatrix} 3^n & 3^n - 2^n \\ 0 & 2^n \end{pmatrix}$

(a)

eigenvalues 3 and 2 A1

eigenvectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

A1A1

$$\mathbf{P} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix},$$
$$\mathbf{D} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$$ A1

(b)

$\mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$ M1

$$\begin{pmatrix} 3^n & 3^n - 2^n \\ 0 & 2^n \end{pmatrix}$$ A2

Every solution is also on ibmathrevision.com/hlai/matrices-exam-practice
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