IB MATHS AI HL · UNIT 1

# Matrices, Determinants and Inverses: exam practice

10 questions · 38 marks · worked solutions and markschemes

Name _____ Date _____ Score / 38

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/matrices-determinants-and-inverses-exam-practice

**1.** **2 marks · Paper 1 style · calculator allowed**

Let $\mathbf{A} = \begin{pmatrix} 1 & 3 \\ -2 & 0 \end{pmatrix}$.

Find $\mathbf{A}^T$ and $\mathbf{A} + \mathbf{A}^T$. **[2]**

**2.** **3 marks · Paper 1 style · calculator allowed**

A shop sells three products. Weekly sales (rows: weeks 1, 2; columns: products) are $\mathbf{S} = \begin{pmatrix} 12 & 8 & 5 \\ 10 & 15 & 6 \end{pmatrix}$ and prices (dollars) are $\mathbf{p} = \begin{pmatrix} 4 \\ 6 \\ 10 \end{pmatrix}$.

Find $\mathbf{Sp}$ and interpret it. **[3]**

**3.** 3 marks · Paper 2 style · calculator allowed

Use an inverse matrix to solve $2x + y - z = 3$, $x - y + 2z = 5$, $3x + 2y + z = 10$.

Find $x$, $y$ and $z$. **[3]**

**4.** 3 marks · Paper 1 style · calculator allowed

Let $\mathbf{A} = \begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix}$.

Find $\det \mathbf{A}$ and $\mathbf{A}^{-1}$. **[3]**

**5.** 3 marks · Paper 1 style · calculator allowed

$\mathbf{A}$ and $\mathbf{B}$ are $2 \times 2$ matrices with $\det \mathbf{A} = 3$ and $\det \mathbf{B} = -2$.

Find $\det(\mathbf{AB})$, $\det(\mathbf{A}^{-1})$ and $\det(2\mathbf{A})$. **[3]**

**6.** 4 marks · Paper 1 style · calculator allowed

Let $\mathbf{A} = \begin{pmatrix} a & 2 \\ 1 & b \end{pmatrix}$ with $\mathbf{A}^2 = \begin{pmatrix} 11 & 10 \\ 5 & 6 \end{pmatrix}$ and $a, b > 0$.

Find $a$ and $b$. **[4]**

**7.** **4 marks · Paper 2 style · calculator allowed**

A café sells coffee, tea and cake. Orders cost $\$13.50$ for $2$ coffees, $1$ tea and $1$ cake; $\$12$ for $1$ coffee, $2$ teas and $1$ cake; $\$19.50$ for $2$ coffees, $1$ tea and $3$ cakes.

Write the system as a matrix equation and solve it. **[4]**

**8.** **3 marks · Paper 2 style · calculator allowed**

Use an inverse matrix to solve the system

$$\begin{aligned} 2x + y - z &= 3 \\ x - 3y + 2z &= 11 \\ 3x + 2y + z &= 6 \end{aligned}$$

**9.** **6 marks · Paper 1 style · calculator allowed**

Consider the transformation matrix $T = \begin{pmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{pmatrix}$, which models the yearly transition of populations between a city and the surrounding suburbs.

The initial population vector in year 0 is $V_0 = \begin{pmatrix} 150 \\ 50 \end{pmatrix}$, where the values represent thousands of people.

The population in year $n$ is given by $V_n = T^n V_0$.

**(a)** Calculate the exact population vector for year 1, $V_1$. **[2 marks]**

**(b)** Use your GDC to calculate the population vector for year 10, $V_{10}$. Give your answers correct to three significant figures. **[4 marks]**

**10.** 7 marks · Paper 1 style · calculator allowed

Let $A$ be a $2 \times 2$ matrix given by $A = \begin{pmatrix} k & 3 \\ 1 & k+2 \end{pmatrix}$, where $k$ is a real number.

a. Find $\det(A)$ in terms of $k$.

b. Find the values of $k$ for which $A^{-1}$ does not exist.

c. Given that $\det(A^{\mathrm{T}}A) = 25$, find the possible values of $k$.

## Answers and worked solutions

### Question 1 — 2 marks

**Answer:** a proof: the worked solution above is the full answer

$\begin{pmatrix}1 & -2\\ 3 & 0\end{pmatrix}$ A1

$\begin{pmatrix}2 & 1\\ 1 & 0\end{pmatrix}$ A1

### Question 2 — 3 marks

**Answer:** $\begin{pmatrix}146\\ 190\end{pmatrix}$: weekly takings

$\begin{pmatrix}146\\ 190\end{pmatrix}$ A2

the takings in weeks 1 and 2 A1

### Question 3 — 3 marks

**Answer:** $x = 2$, $y = 1$, $z = 2$

$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$ M1

$x = 2$, $y = 1$, $z = 2$ A2

### Question 4 — 3 marks

**Answer:** $10$

$\det \mathbf{A} = 10$ A1

$\frac{1}{10}\begin{pmatrix}1 & 3\\ -2 & 4\end{pmatrix}$ A2

### Question 5 — 3 marks

**Answer:** $-6$, $\frac{1}{3}$, $12$

$-6$ A1

$\frac{1}{3}$ A1

$4 \times 3 = 12$ A1

### Question 6 — 4 marks

**Answer:** $a = 3, b = 2$

$\mathbf{A}^2 = \begin{pmatrix}a^2+2 & 2a+2b\\ a+b & 2+b^2\end{pmatrix}$ M1

A1

$a^2 = 9$, $b^2 = 4$ A1

$a = 3, b = 2$ A1

### Question 7 — 4 marks

**Answer:** coffee $\$4$, tea $\$2.50$, cake $\$3$

$\begin{pmatrix}2 & 1 & 1\\ 1 & 2 & 1\\ 2 & 1 & 3\end{pmatrix}\begin{pmatrix}c\\ t\\ k\end{pmatrix} = \begin{pmatrix}13.5\\ 12\\ 19.5\end{pmatrix}$

M1A1

coffee $\$4$, tea $\$2.50$ A1

cake $\$3$ A1

### Question 8 — 3 marks

**Answer:** $x = 3$; $y = -2$; $z = 1$

$\begin{pmatrix}2 & 1 & -1\\ 1 & -3 & 2\\ 3 & 2 & 1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}3\\ 11\\ 6\end{pmatrix}$

(M1)

$\boldsymbol{x} = A^{-1}\boldsymbol{b}$ (M1)

$x = 3,\ y = -2,\ z = 1$ A1

### Question 9 — 6 marks

**Answer:** (a) $(125, 75)$; (b) $(69.0, 131)$

**(a)**

$V_1 = TV_0 = \begin{pmatrix}0.8 & 0.1\\ 0.2 & 0.9\end{pmatrix}\begin{pmatrix}150\\ 50\end{pmatrix}$

[M1]

$V_1 = \begin{pmatrix}0.8(150)+0.1(50)\\ 0.2(150)+0.9(50)\end{pmatrix} = \begin{pmatrix}120+5\\ 30+45\end{pmatrix} = \begin{pmatrix}125\\ 75\end{pmatrix}$

[A1]

**(b)** Use $V_{10} = T^{10}V_0$ [M1]

On the GDC (matrix mode), enter $T$ and $V_0$ and evaluate $T^{10}$:

$T^{10} \approx \begin{pmatrix}0.352165 & 0.323917\\ 0.647835 & 0.676083\end{pmatrix}$

[(A1)]

Multiply by $V_0$:

$V_{10} = \begin{pmatrix}69.0206\ldots\\ 130.979\ldots\end{pmatrix}$

$V_{10} \approx \begin{pmatrix}69.0\\ 131\end{pmatrix}$ (thousands, 3 s.f.)

[A1][A1]

*(Check: $69.0 + 131 = 200$, the total population is conserved.)*

### Question 10 — 7 marks

**Answer:** (a) $k^2 + 2k - 3$; (b) $-3, 1$; (c) $-4, 2$

**(a)**

Use $\det\begin{pmatrix}a & b\\ c & d\end{pmatrix} = ad - bc$:

$\det(A) = k(k+2) - 3(1)$ M1

$\det(A) = k^2 + 2k - 3$ A1

**(b)**

$A^{-1}$ does not exist when $\det(A) = 0$:

$k^2 + 2k - 3 = (k+3)(k-1) = 0$

$k = -3$ or $k = 1$ A1

**(c)**

Use $\det(A^{\mathrm{T}}A) = \det(A^{\mathrm{T}})\det(A) = (\det A)^2$:

$(\det A)^2 = 25$ M1

$\det A = \pm 5$ A1

Case $\det A = 5$:

$k^2 + 2k - 8 = (k+4)(k-2) = 0 \implies k = -4$ or $k = 2$ A1

Case $\det A = -5$:

$k^2 + 2k + 2 = 0$ has discriminant $2^2 - 4(1)(2) = -4 < 0$, so no real solutions R1

The possible values are $k = -4$ and $k = 2$.

Every solution is also on ibmathrevision.com/hlai/matrices-determinants-and-inverses-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.