Logarithms and Exponentials: exam practice

10 questions · 31 marks · worked solutions and markschemes

Name _____ Date _____ Score / 31

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/logarithms-and-exponentials-exam-practice

1. **2 marks · Paper 2 style · calculator allowed**

The number of users of an app doubles every 9 months. There are $12\,000$ users now.

Write a model $U = 12000 \times 2^{\frac{t}{9}}$ and find the number of users after 2 years. **[2]**

2. **2 marks · Paper 1 style · calculator allowed**

Solve $e^{2x} = 7e^{x}$.

Find x. **[2]**

3. **3 marks · Paper 1 style · calculator allowed**

Solve $5 \times 3^{x} = 200$.

Find x. **[3]**

4. **3 marks · Paper 2 style · calculator allowed**

The value of a rare coin is $V = 150 \times 1.08^{t}$ dollars, t years after 2020.

Find the year in which the value first exceeds $\$500$. **[3]**

5. **3 marks · Paper 1 style · calculator allowed**

Given $\ln a = 2.4$ and $\ln b = 0.9$, find the value of $\ln\left(\dfrac{a^2}{\sqrt{b}}\right)$ and of $\ln(ab)^3$.

Find both values. **[3]**

6. **3 marks · Paper 1 style · calculator allowed**

Solve $\log_2(x+6) - \log_2(x-1) = 3$.

Find x. **[3]**

7. **3 marks · Paper 2 style · calculator allowed**

A radioactive sample has a half-life of 5730 years.

Find the percentage of the sample left after 2000 years. **[3]**

8. **4 marks · Paper 2 style · calculator allowed**

A population of bacteria is modelled by $N(t) = 800e^{0.15t}$, where t is in hours.

(a) Find the population after 5 hours. **[2]**

(b) Find the time taken for the population to triple. **[2]**

9. **4 marks · Paper 1 style · calculator allowed**

Write $2\log x + \log 5 - \log 2$ as a single logarithm, and solve $2\log x + \log 5 - \log 2 = 1$, where $\log$ is base 10.

(a) Single logarithm. **[2]**

(b) Solve the equation. **[2]**

10. **4 marks · Paper 2 style · calculator allowed**

The pH of a solution is $\text{pH} = -\log_{10}[\text{H}^+]$.

(a) Find the pH when $[\text{H}^+] = 3.2 \times 10^{-5}$. **[2]**

(b) Find $[\text{H}^+]$ when the pH is 8.2. **[2]**

Answers and worked solutions

Question 1 — 2 marks

Answer: $76\,200$

$t = 24$ months: $12000 \times 2^{\frac{24}{9}}$ **M1**

$76\,200$ **A1**

Question 2 — 2 marks

Answer: $x = \ln 7$

$e^x = 7$ (as $e^x \neq 0$) **M1**

$x = \ln 7$ **A1**

Question 3 — 3 marks

Answer: $x = 3.36$

$3^x = 40$ **A1**

$x = \dfrac{\ln 40}{\ln 3}$ **M1**

$x = 3.36$ **A1**

Question 4 — 3 marks

Answer: 2036

$1.08^t > \dfrac{10}{3}$ **M1**

$t > 15.6$ **A1**

during 2036 **A1**

Question 5 — 3 marks

Answer: 4.35; 9.9

$2(2.4) - \dfrac{1}{2}(0.9)$ **M1**

4.35 **A1**

$3(2.4 + 0.9) = 9.9$ **A1**

Question 6 — 3 marks

Answer: 2

$\dfrac{x+6}{x-1} = 8$ **M1**

$7x = 14$ **A1**

$x = 2$ **A1**

Question 7 — 3 marks

Answer: 78.5%

$m = m_0\left(\dfrac{1}{2}\right)^{\frac{t}{5730}}$ **M1**

$\left(\dfrac{1}{2}\right)^{\frac{2000}{5730}}$ **A1**

78.5% **A1**

Question 8 — 4 marks

Answer: (a) 1690; (b) 7.32 hours

(a)

$800e^{0.75}$ **M1**

1690 **A1**

(b)

$e^{0.15t} = 3$ **M1**

7.32 hours **A1**

Question 9 — 4 marks

Answer: (a) $\log \dfrac{5x^2}{2}$; (b) 2

(a)

$\log \dfrac{5x^2}{2}$ **A2**

(b)

$\dfrac{5x^2}{2} = 10$ **M1**

$x = 2$ **A1**

Question 10 — 4 marks

Answer: (a) 4.49; (b) 6.31×10^{-9}

(a)

$-\log_{10}(3.2 \times 10^{-5})$ **M1**

4.49 **A1**

(b)

$10^{-8.2}$ **M1**

6.31×10^{-9} **A1**

Every solution is also on ibmathrevision.com/hlai/logarithms-and-exponentials-exam-practice
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