IB MATHS AI HL · UNIT 5

Limits, Derivatives & Rates of Change: exam practice

10 questions · 28 marks · worked solutions and markschemes

Name _____ Date _____ Score / 28

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/limits-derivatives-and-rates-of-change-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

The height of a plant is $h(t) = 2 + 0.5t^2$ cm after t days.

Find the average rate of growth from $t = 2$ to $t = 4$, and the instantaneous rate at $t = 3$. **[3]**

2. **2 marks · Paper 1 style · calculator allowed**

Find $\lim_{x \to \infty} \dfrac{4x^2 - 1}{2x^2 + 3x}$.

Find the limit. **[2]**

3. **2 marks · Paper 1 style · calculator allowed**

The number of bacteria is $N(t) = 200e^{0.3t}$.

Find the rate of growth at $t = 5$ and interpret it. **[2]**

4. 2 marks · Paper 1 style · calculator allowed

Find $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$ and explain why the function is not defined at $x = 2$.

Find the limit. **[2]**

5. 3 marks · Paper 1 style · calculator allowed

A chord on $y = x^2$ joins $(3, 9)$ and $(3 + h, (3 + h)^2)$.

Find the gradient of the chord in terms of h and its limit as $h \to 0$. **[3]**

6. 3 marks · Paper 1 style · calculator allowed

Let $f(x) = x^2 - 6x + 10$.

Find the coordinates of the minimum point and justify that it is a minimum. **[3]**

7. 3 marks · Paper 2 style · calculator allowed

Water leaks from a tank so that the volume is $V(t) = 500 - 20t + 0.2t^2$ litres for $0 \le t \le 50$ minutes.

Find the rate of change at $t = 10$ and the time when the tank stops losing water. **[3]**

8. **3 marks · Paper 2 style · calculator allowed**

Let $f(x) = xe^{-0.5x}$ for $x \geq 0$.

Find the coordinates of the maximum. **[3]**

9. **3 marks · Paper 1 style · calculator allowed**

The displacement of a particle is $s(t) = t^3$ m.

Find the average velocity from $t = 1$ to $t = 3$, and the instantaneous velocity at $t = 2$. **[3]**

10. **4 marks · Paper 2 style · calculator allowed**

The profit of a company is $P(x) = -x^3 + 12x^2 - 36x + 100$ thousand dollars for x thousand units, $0 \leq x \leq 10$.

Find the local maximum and local minimum of P. **[4]**

Answers and worked solutions

Question 1 — 3 marks

Answer: 3 cm/day; 3 cm/day

$\frac{10-4}{2} = 3$ cm/day **A1**

$h'(t) = t$ **M1**

3 cm/day **A1**

Question 2 — 2 marks

Answer: 2

divide by x^2 **M1**

2 **A1**

Question 3 — 2 marks

Answer: 269 per hour

$N'(t) = 60e^{0.3t}$ **M1**

269 bacteria per hour **A1**

Question 4 — 2 marks

Answer: 4

$x + 2 \to 4$ **A1**

division by zero at $x = 2$ **R1**

Question 5 — 3 marks

Answer: $6 + h$; 6

$\frac{(3+h)^2 - 9}{h}$ **M1**

$6 + h$ **A1**

6 **A1**

Question 6 — 3 marks

Answer: $(3, 1)$

$f'(x) = 2x - 6 = 0$ **M1**

$(3,\ 1)$ **A1**

$f''(x) = 2 > 0$ **R1**

Question 7 — 3 marks

Answer: -16 litres/min; $t = 50$

$V'(t) = -20 + 0.4t$ **A1**

-16 litres/min **A1**

$t = 50$ min **A1**

Question 8 — 3 marks

Answer: $(2,\ 0.736)$

$f'(x) = (1 - 0.5x)e^{-0.5x}$ **A1**

$x = 2$ **A1**

$(2,\ 0.736)$ **A1**

Question 9 — 3 marks

Answer: $13\ \text{m/s}$; $12\ \text{m/s}$

$\frac{27-1}{2} = 13\ \text{m/s}$ **A1**

$v = 3t^2$ **M1**

$12\ \text{m/s}$ **A1**

Question 10 — 4 marks

Answer: min $(2, 68)$; max $(6, 100)$

$P'(x) = -3x^2 + 24x - 36 = 0$ **M1**

$x = 2$, $x = 6$ **A1**

minimum $(2,\ 68)$ **A1**

maximum $(6,\ 100)$ **A1**

Every solution is also on ibmathrevision.com/hlai/limits-derivatives-and-rates-of-change-exam-practice
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