IB MATHS AI HL · UNIT 5

# Integration: exam practice

10 questions · 35 marks · worked solutions and markschemes

Name _____ Date _____ Score / 35

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/integration-exam-practice

**1.** **3 marks · Paper 1 style · calculator allowed**

Find the area enclosed by $y = 6x - x^2$ and the $x$-axis.

Find the area. **[3]**

**2.** **3 marks · Paper 2 style · calculator allowed**

Use the trapezium rule with $4$ intervals to estimate $\int_0^2 e^{x^2}\,dx$.

Find the estimate. **[3]**

**3.** **3 marks · Paper 2 style · calculator allowed**

A function has $f'(x) = 3x^2 - 4x + 1$ and $f(2) = 5$.

Find $f(x)$. **[3]**

**4.** **3 marks · Paper 2 style · calculator allowed**

The speed of a car (m/s) is recorded every $10$ s: $0,\ 8,\ 14,\ 17,\ 18$ at $t = 0, 10, 20, 30, 40$.

Use the trapezium rule to estimate the distance travelled in $40$ s. **[3]**

**5.** **3 marks · Paper 1 style · calculator allowed**

Find the area enclosed by $y = x^2$ and $y = 4x$.

Find the area. **[3]**

**6.** **3 marks · Paper 2 style · calculator allowed**

Use the trapezium rule with $5$ strips to estimate $\int_1^6 \ln x \, dx$, and state whether it is an overestimate or underestimate.

Find the estimate. **[3]**

**7.** **4 marks · Paper 1 style · calculator allowed**

The region enclosed by the curve $y = \frac{1}{x}$, the $x$-axis, and the vertical lines $x = 1$ and $x = 3$ is rotated $360°$ around the $x$-axis.

Calculate the exact volume of the solid generated.

**8.** **4 marks · Paper 1 style · calculator allowed**

Use the substitution $u = x^3 + 1$ to find the indefinite integral $\int \frac{x^2}{x^3+1}\,dx$.

**9.** **4 marks · Paper 1 style · calculator allowed**

Find the exact value of the indefinite integral $\int \frac{5}{2x-3}\,dx$.

**10.** **5 marks · Paper 1 style · calculator allowed**

Use the substitution $u = x^2 + 1$ to evaluate exactly the definite integral $\int_0^2 2xe^{x^2+1}\,dx$.

## Answers and worked solutions

### Question 1

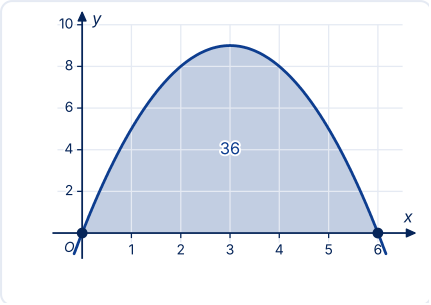
3 marks

**Answer:** $36$

roots $0$ and $6$ **A1**

$\int_0^6 (6x - x^2)\,dx$ **M1**

$36$ **A1**




Area $= \int_0^6 (6x - x^2)\,\mathrm{d}x = 36$

### Question 2

3 marks

**Answer:** $20.6$

$h = 0.5$ **A1**

$\frac{0.5}{2}\left[1 + e^4 + 2\left(e^{0.25} + e + e^{2.25}\right)\right]$ **M1**

$20.6$ **A1**

### Question 3

3 marks

**Answer:**

$f(x) = x^3 - 2x^2 + x + 3$

$x^3 - 2x^2 + x + c$ **A1**

$8 - 8 + 2 + c = 5$ **M1**

$f(x) = x^3 - 2x^2 + x + 3$ **A1**

### Question 4

3 marks

**Answer:** $480\ m$

$\frac{10}{2}[0 + 18 + 2(8 + 14 + 17)]$ **M1A1**

$480\ m$ **A1**

### Question 5

3 marks

**Answer:** $\frac{32}{3}$

meet at $x = 0$, $x = 4$ **A1**

$\int_0^4 (4x - x^2)\,dx$ **M1**

$\frac{32}{3}$ **A1**

### Question 6

3 marks

**Answer:** $5.68$, an underestimate

$\frac{1}{2}[0 + \ln 6 + 2(\ln 2 + \ln 3 + \ln 4 + \ln 5)]$ **M1A1**

$5.68$; underestimate (curve concave down) **A1**

### Question 7

4 marks

**Answer:** $2\pi/3$

Volume $V = \pi \int_1^3 y^2\,dx$. **[M1]**

$V = \pi \int_1^3 \left(\frac{1}{x}\right)^2 dx = \pi \int_1^3 x^{-2}\,dx$ . **[A1]**

$= \pi\left[-x^{-1}\right]_1^3 = \pi\left[-\frac{1}{x}\right]_1^3$. **[M1]**

$= \pi\left(-\frac{1}{3} - (-1)\right) = \pi\left(1 - \frac{1}{3}\right) = \frac{2\pi}{3}$ . **[A1]**

### Question 8

4 marks

**Answer:** $\ln|x^3 + 1|/3 + C$

Let $u = x^3 + 1$. Then

$\frac{du}{dx} = 3x^2 \implies dx = \frac{du}{3x^2}$ **M1**

Substitute: the integral becomes

$\int \frac{x^2}{u} \cdot \frac{du}{3x^2}$ **M1**

$= \frac{1}{3}\int \frac{1}{u}\,du$ **A1**

Integrate and substitute back:

$= \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|x^3 + 1| + C$ **A1**

### Question 9

4 marks

**Answer:** $(5/2)\ln|2x - 3| + C$

Let $u = 2x - 3$, then

$\frac{du}{dx} = 2 \implies dx = \frac{du}{2}$ **M1**

Substitute:

$\int \frac{5}{u} \cdot \frac{du}{2} = \frac{5}{2}\int \frac{1}{u}\,du$ **M1**

Integrate: $= \frac{5}{2}\ln|u| + C$ **A1**

Substitute back:

$= \frac{5}{2}\ln|2x - 3| + C$ **A1**

### Question 10

5 marks

**Answer:** $e^5 - e$

Let $u = x^2 + 1$. Then

$\frac{du}{dx} = 2x \implies dx = \frac{du}{2x}$. **[M1]**

Change limits: when

$x = 0, u = 1$. When

$x = 2, u = 5$. **[M1]**

Integral becomes

$\int_1^5 2xe^u \frac{du}{2x} = \int_1^5 e^u\,du$. **[A1]**

$= \left[e^u\right]_1^5$ **[M1]**

$= e^5 - e^1$. **[A1]**

Every solution is also on ibmathrevision.com/hlai/integration-exam-practice
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