Functions: exam practice

10 questions · 29 marks · worked solutions and markschemes

Name _____ Date _____ Score / 29

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/functions-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

Let $f(x) = 2x^2 - 1$ and $g(x) = x + 3$.

Find $(f \circ g)(-1)$ and $(g \circ f)(x)$. **[3]**

2. **2 marks · Paper 1 style · calculator allowed**

Let $f(x) = 2x - 5$ and $g(x) = x^2 + 1$.

Find an expression for the composite function $(g \circ f)(x)$.

3. **2 marks · Paper 1 style · calculator allowed**

Let $f(x) = 4 - x^2$ with domain $-1 \leq x \leq 3$.

Find the range of f. **[2]**

4. **3 marks · Paper 1 style · calculator allowed**

Let $f(x) = \sqrt{x+3}$ for $x \geq -3$.

Find $f^{-1}(x)$ and state its domain. **[3]**

5. **3 marks · Paper 2 style · calculator allowed**

For the data $x = 1, 2, 3, 4$ and $y = 2.0,\ 5.7,\ 10.4,\ 16.0$, a model $y = ax^n$ is proposed.

Use a regression of $\ln y$ on $\ln x$ to find a and n. **[3]**

6. **3 marks · Paper 1 style · calculator allowed**

Let $h(x) = \dfrac{2}{x-4}$, $x \neq 4$.

State the range of h, and find $h^{-1}(x)$. **[3]**

7. **3 marks · Paper 2 style · calculator allowed**

A model $y = a\,b^x$ is fitted to data. The regression line of $\ln y$ on x is $\ln y = 0.405x + 1.10$.

Find a and b. **[3]**

8. **3 marks · Paper 1 style · calculator allowed**

Let $f(x) = x^2 + 2$ for $x \geq 0$ and $g(x) = \ln x$ for $x > 0$.

Find $(g \circ f)(x)$ and its range. **[3]**

9. **3 marks · Paper 1 style · calculator allowed**

Let $f(x) = \dfrac{x+2}{3}$.

Solve $f^{-1}(x) = f(x)$. **[3]**

10. **4 marks · Paper 2 style · calculator allowed**

The data below are thought to follow $y = ae^{bx}$.

x	0	2	4	6
y	5.0	8.1	13.4	22.0

Find the regression line of $\ln y$ on x and hence a and b. **[4]**

Answers and worked solutions

Question 1 — 3 marks

Answer: 7; $2x^2 + 2$

$f(2) = 7$ **A1**

$2x^2 - 1 + 3$ **M1**

$2x^2 + 2$ **A1**

Question 2 — 2 marks

Answer: $4x^2 - 20x + 26$

Substitute $f(x)$ into g:

$(g \circ f)(x) = g(2x - 5) = (2x - 5)^2 + 1$ **M1**

Expand:

$= 4x^2 - 20x + 25 + 1 = 4x^2 - 20x + 26$ **A1**

Question 3 — 2 marks

Answer: $-5 \leq f(x) \leq 4$

maximum 4 at $x = 0$ **A1**

$-5 \leq f(x) \leq 4$ **A1**

Question 4 — 3 marks

Answer: $f^{-1}(x) = x^2 - 3$, $x \geq 0$

$x = y^2 - 3$ **M1**

$f^{-1}(x) = x^2 - 3$ **A1**

$x \geq 0$ **A1**

Question 5 — 3 marks

Answer: $a = 2.00$, $n = 1.50$

GDC: $\ln y = 1.50 \ln x + 0.696$ **M1**

$n = 1.50$ **A1**

$a = 2.00$ **A1**

Question 6 — 3 marks

Answer: range $h(x) \neq 0$;

$h^{-1}(x) = \dfrac{2}{x} + 4$

$h(x) \neq 0$ **A1**

$x - 4 = \dfrac{2}{y}$ **M1**

$h^{-1}(x) = \dfrac{2}{x} + 4$ **A1**

Question 7 — 3 marks

Answer: $a = 3.00$, $b = 1.50$

$\ln y = \ln a + x \ln b$ **M1**

$a = e^{1.10} = 3.00$ **A1**

$b = e^{0.405} = 1.50$ **A1**

Question 8 — 3 marks

Answer: $\ln(x^2 + 2)$; $\geq \ln 2$

$\ln(x^2 + 2)$ **A1**

$x^2 + 2 \geq 2$ **M1**

$(g \circ f)(x) \geq \ln 2$ **A1**

Question 9 — 3 marks

Answer: 1

$f^{-1}(x) = 3x - 2$ **A1**

$3x - 2 = \dfrac{x + 2}{3}$ **M1**

$x = 1$ **A1**

Question 10 — 4 marks

Answer: $a = 4.98$, $b = 0.247$

$\ln y = 0.247x + 1.61$ $M1$**A1**

$a = 4.98$ **A1**

$b = 0.247$ **A1**

Every solution is also on ibmathrevision.com/hlai/functions-exam-practice
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