Functions: Domain, Range & Inverses: exam practice

10 questions · 29 marks · worked solutions and markschemes

Name _____ Date _____ Score / 29

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/functions-domain-range-and-inverses-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

The cost of hiring a bike is $C(h) = 8 + 3.5h$ dollars for h hours, $1 \leq h \leq 8$.

Find the range of C and the inverse function. **[3]**

2. **2 marks · Paper 2 style · calculator allowed**

Let $f(x) = x^3 + x$.

Use technology to find $f^{-1}(10)$ and $f^{-1}(3)$. **[2]**

3. **2 marks · Paper 1 style · calculator allowed**

The graph of $y = f(x)$ passes through $(1, 4)$ and $(3, 9)$, and f is one-to-one.

Write down $f^{-1}(9)$, and the point on $y = f^{-1}(x)$ that corresponds to $(1, 4)$. **[2]**

4. 3 marks · Paper 1 style · calculator allowed

Let $f(x) = \ln(2x - 6)$.

State the domain of f and find $f^{-1}(x)$. **[3]**

5. 3 marks · Paper 1 style · calculator allowed

Let $g(x) = \dfrac{1}{\sqrt{9 - x}}$.

Find the largest possible domain and the range of g. **[3]**

6. 3 marks · Paper 1 style · calculator allowed

Let $f(x) = e^x + 1$ and $g(x) = 2x$.

Find $(f \circ g)^{-1}(x)$. **[3]**

7. 3 marks · Paper 1 style · calculator allowed

Let $f(x) = \dfrac{3x}{x + 1}$, $x \neq -1$.

Find $f^{-1}(x)$ and the value of x for which $f^{-1}(x) = 2$. **[3]**

8. 3 marks · Paper 1 style · calculator allowed

Let $f(x) = \frac{x}{2} + 1$ and $g(x) = x^2$.

Solve $(f \circ g)(x) = (g \circ f)(x)$. **[3]**

9. 4 marks · Paper 1 style · calculator allowed

Let $f(x) = 5 - 2x$ for $-1 \leq x \leq 4$.

Find the range of f, and $f^{-1}(x)$ with its domain. **[4]**

10. 3 marks · Paper 1 style · calculator allowed

Let $f(x) = (x - 2)^2 + 1$ for $x \geq k$. f has an inverse.

Find the least value of k, and $f^{-1}(x)$ for this k. **[3]**

Answers and worked solutions

Question 1 — 3 marks

Answer: $11.5 \leq C \leq 36$; $C^{-1}(x) = \dfrac{x-8}{3.5}$

$11.5 \leq C \leq 36$ **A1**

$h = \dfrac{C-8}{3.5}$ $M1$**A1**

Question 2 — 2 marks

Answer: 2; 1.21

$x^3 + x = 10$: $x = 2$ **A1**

$x^3 + x = 3$: $x = 1.21$ **A1**

Question 3 — 2 marks

Answer: 3; $(4, 1)$

3 **A1**

$(4, 1)$ **A1**

Question 4 — 3 marks

Answer: $x > 3$; $f^{-1}(x) = \dfrac{e^x + 6}{2}$

$x > 3$ **A1**

$2x - 6 = e^y$ **M1**

$f^{-1}(x) = \dfrac{e^x + 6}{2}$ **A1**

Question 5 — 3 marks

Answer: $x < 9$; $g(x) > 0$

$9 - x > 0$ **M1**

$x < 9$ **A1**

$g(x) > 0$ **A1**

Question 6 — 3 marks

Answer: $\dfrac{1}{2}\ln(x-1)$

$(f \circ g)(x) = e^{2x} + 1$ **A1**

$x = \dfrac{1}{2}\ln(y-1)$ **M1**

$(f \circ g)^{-1}(x) = \dfrac{1}{2}\ln(x-1)$ **A1**

Question 7 — 3 marks

Answer: $f^{-1}(x) = \dfrac{x}{3-x}$; $x = 2$

$f^{-1}(x) = \dfrac{x}{3-x}$ **A2**

$x = f(2) = 2$ **A1**

Question 8 — 3 marks

Answer: $x = 0$, $x = 4$

$\dfrac{x^2}{2} + 1 = \left(\dfrac{x}{2} + 1\right)^2$ **M1**

$\dfrac{x^2}{4} - x = 0$ **A1**

$x = 0$, $x = 4$ **A1**

Question 9 — 4 marks

Answer: $f^{-1}(x) = \dfrac{5-x}{2}$, $-3 \leq x \leq 7$

$-3 \leq f(x) \leq 7$ **A1**

$x = \dfrac{5-y}{2}$ **M1**

$f^{-1}(x) = \dfrac{5-x}{2}$ **A1**

$-3 \leq x \leq 7$ **A1**

Question 10 — 3 marks

Answer: $k = 2$; $f^{-1}(x) = 2 + \sqrt{x-1}$

$k = 2$ (vertex) **A1**

$x = 2 + \sqrt{y-1}$ **M1**

$f^{-1}(x) = 2 + \sqrt{x-1}$, $x \geq 1$ **A1**

Every solution is also on ibmathrevision.com/hlai/functions-domain-range-and-inverses-exam-practice
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