Exponential Modelling: exam practice

10 questions · 33 marks · worked solutions and markschemes

Name _____ Date _____ Score / 33

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/exponential-modelling-exam-practice

1. **2 marks · Paper 1 style · calculator allowed**

The model $A = A_0 e^{-0.03t}$ gives the area of a melting ice sheet.

Find the percentage of the area left after 20 years. **[2]**

2. **3 marks · Paper 2 style · calculator allowed**

An investment grows as $V = a \times b^t$. It is worth $\$4000$ at $t = 0$ and $\$4840$ at $t = 2$.

Find a and b, and the value at $t = 5$. **[3]**

3. **3 marks · Paper 1 style · calculator allowed**

A population doubles every 6 years. It is 3500 now.

Write a model and find the population in 15 years. **[3]**

4. **3 marks · Paper 2 style · calculator allowed**

The value of a laptop falls by 22% each year. It costs $\$1800$ new.

Find after how many complete years its value is first below $\$500$. **[3]**

5. **3 marks · Paper 2 style · calculator allowed**

The number of followers of a channel is $F = 800e^{0.12t}$, t in weeks.

Find the rate of growth per week as a percentage, and the number of weeks for F to reach $10\,000$. **[3]**

6. **3 marks · Paper 2 style · calculator allowed**

Water drains from a tank so that the depth is $d = 4 \times 0.9^{t}$ m after t minutes.

Find the depth after 8 minutes and the time when the depth is 1 m. **[3]**

7. **4 marks · Paper 2 style · calculator allowed**

A body cools from 90 °C in a room at 20 °C, following $T = 20 + 70e^{-kt}$. After 10 minutes it is 55 °C.

Find k and the temperature after 25 minutes. **[4]**

8. **4 marks · Paper 2 style · calculator allowed**

An exponential model $N = ae^{kt}$ passes through $(0, 120)$ and $(5, 300)$.

Find a and k, and predict N at $t = 8$. **[4]**

9. **4 marks · Paper 2 style · calculator allowed**

A radioactive isotope has a half-life of 8 days. A sample has mass 60 g.

Find the decay constant k in $m = 60e^{-kt}$, and the time for the mass to fall to 5 g. **[4]**

10. **4 marks · Paper 2 style · calculator allowed**

A drug in the bloodstream decays as $D(t) = 250e^{-0.18t}$ mg, t hours after an injection.

(a) Find the amount after 5 hours. **[2]**

(b) Find the half-life of the drug. **[2]**

Answers and worked solutions

Question 1 2 marks

Answer: 54.9%

$e^{-0.6}$ M1

54.9% A1

Question 2 3 marks

Answer: $a = 4000$, $b = 1.1$; $\$6442.04$

$a = 4000$ A1

$b^2 = 1.21$, $b = 1.1$ A1

$\$6442.04$ A1

Question 3 3 marks

Answer: $19\,800$

$P = 3500 \times 2^{\frac{t}{6}}$ A1

$3500 \times 2^{2.5}$ M1

$19\,800$ A1

Question 4 3 marks

Answer: 6

$1800(0.78)^n < 500$ M1

$n > 5.15$ A1

6 years A1

Question 5 3 marks

Answer: 12.7%; 21.0 weeks

$e^{0.12} - 1 = 12.7\%$ A1

$e^{0.12t} = 12.5$ M1

21.0 weeks A1

Question 6 3 marks

Answer: $1.72\,m$; 13.2 minutes

$4(0.9)^8 = 1.72\,m$ A1

$0.9^t = 0.25$ M1

13.2 min A1

Question 7 4 marks

Answer: $k = 0.0693$; $32.4\,°C$

$35 = 70e^{-10k}$ M1

$k = 0.0693$ A1

$20 + 70e^{-25k}$ M1

$32.4\,°C$ A1

Question 8 4 marks

Answer: $a = 120$, $k = 0.183$; 520

$a = 120$ A1

$e^{5k} = 2.5$, $k = 0.183$ $M1$A1

$N(8) = 520$ A1

Question 9 4 marks

Answer: $k = 0.0866$; 28.7 days

$e^{-8k} = 0.5$ M1

$k = 0.0866$ A1

$60e^{-kt} = 5$ M1

28.7 days A1

Question 10 4 marks

Answer: (a) 102 mg; (b) 3.85 hours

(a)

$250e^{-0.9}$ M1

102 mg A1

(b)

$e^{-0.18t} = 0.5$ M1

3.85 hours A1

Every solution is also on ibmathrevision.com/hlai/exponential-modelling-exam-practice
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