Differentiation: exam practice

10 questions · 30 marks · worked solutions and markschemes

Name _____ Date _____ Score / 30

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/differentiation-exam-practice

1. **2 marks · Paper 2 style · calculator allowed**

The side of a square increases at 2 cm/s.

Find the rate of increase of its area when the side is 15 cm. **[2]**

2. **2 marks · Paper 1 style · calculator allowed**

Differentiate $y = e^{3x^2}$ and $y = \ln(5x - 2)$.

Find both derivatives. **[2]**

3. **2 marks · Paper 2 style · calculator allowed**

Air is pumped into a spherical ball so that the radius increases at 0.2 cm/s.

Find the rate of increase of the surface area when $r = 10$ cm. **[2]**

4. 2 marks · Paper 1 style · calculator allowed

Differentiate $y = \sin^3(2x)$.

Find $\frac{dy}{dx}$. **[2]**

5. 3 marks · Paper 2 style · calculator allowed

The cost of producing x units is $C(x) = 0.02x^2 - 4x + 500$.

Find the number of units that minimises cost and the minimum cost. **[3]**

6. 3 marks · Paper 1 style · calculator allowed

Let $f(x) = x^3 - 6x^2 + 9x$.

Find the point of inflexion and the interval where f is concave up. **[3]**

7. 3 marks · Paper 1 style · calculator allowed

Let $f(x) = e^x - x^2$.

Find $f''(x)$ and the exact x-coordinate of the point of inflexion. **[3]**

8. **4 marks · Paper 1 style · calculator allowed**

Consider the function $f(x) = x^4 - 6x^2 + 5$.

Find the exact x-coordinates of any points of inflection on the graph of $y = f(x)$.

9. **4 marks · Paper 2 style · calculator allowed**

A 1.8 m tall person walks away from a 6 m lamp post at 1.2 m/s.

Find the rate at which the length of the shadow increases. **[4]**

10. **5 marks · Paper 2 style · calculator allowed**

A spherical balloon is being inflated such that its volume is increasing at a constant rate of $20\ \text{cm}^3\ \text{s}^{-1}$.

Given that the volume of a sphere is $V = \frac{4}{3}\pi r^3$, find the exact rate of change of the radius when the radius is $5\ \text{cm}$.

Answers and worked solutions

Question 1 — 2 marks

Answer: $60\ \text{cm}^2/\text{s}$

$\dfrac{dA}{dt} = 2s\dfrac{ds}{dt}$ **M1**

$60\ \text{cm}^2/\text{s}$ **A1**

Question 2 — 2 marks

Answer: $6xe^{3x^2}$; $\dfrac{5}{5x-2}$

$6xe^{3x^2}$ **A1**

$\dfrac{5}{5x-2}$ **A1**

Question 3 — 2 marks

Answer: $50.3\ \text{cm}^2/\text{s}$

$\dfrac{dS}{dt} = 8\pi r\dfrac{dr}{dt}$ **M1**

$50.3\ \text{cm}^2/\text{s}$ **A1**

Question 4 — 2 marks

Answer: $6\sin^2(2x)\cos(2x)$

chain rule twice **M1**

$6\sin^2(2x)\cos(2x)$ **A1**

Question 5 — 3 marks

Answer: 100 units; 300

$C'(x) = 0.04x - 4 = 0$ **M1**

$x = 100$ **A1**

300 **A1**

Question 6 — 3 marks

Answer: $(2, 2)$; $x > 2$

$f''(x) = 6x - 12$ **A1**

$(2,\ 2)$ **A1**

$x > 2$ **A1**

Question 7 — 3 marks

Answer: $x = \ln 2$

$f''(x) = e^x - 2$ **A1**

$e^x = 2$ **M1**

$x = \ln 2$ **A1**

Question 8 — 4 marks

Answer: $1, -1$

First derivative:

$f'(x) = 4x^3 - 12x$ **A1**

Second derivative:

$f''(x) = 12x^2 - 12$ **A1**

Points of inflection occur where $f''(x) = 0$ and concavity changes:

$12x^2 - 12 = 0 \implies x^2 = 1$ **M1**

$x = 1$ and $x = -1$ **A1**

Check: $f''(0) = -12 < 0$ and $f''(\pm 2) = 36 > 0$, so the concavity changes at both points.

Question 9 — 4 marks

Answer: $0.514\ \text{m/s}$

similar triangles: $\dfrac{s}{1.8} = \dfrac{s+x}{6}$ **M1**

$s = \dfrac{3x}{7}$ **A1**

$\dfrac{ds}{dt} = \dfrac{3}{7}(1.2)$ **M1**

$0.514\ \text{m/s}$ **A1**

Question 10 — 5 marks

Answer: $1/(5\pi)$

Given $\frac{dV}{dt} = 20$.

Differentiate the volume formula: $\frac{dV}{dr} = 4\pi r^2$. **[A1]**

Use the chain rule: $\frac{dV}{dt} = \frac{dV}{dr} \times \frac{dr}{dt}$. **[M1]**

Substitute $\frac{dV}{dt} = 20$: $20 = 4\pi r^2 \times \frac{dr}{dt} \implies \frac{dr}{dt} = \frac{20}{4\pi r^2}$. **[A1]**

Substitute $r = 5$: $\frac{dr}{dt} = \frac{20}{4\pi(25)}$ **[M1]**

$= \frac{20}{100\pi} = \frac{1}{5\pi}\ \text{cm s}^{-1}$. **[A1]**

Every solution is also on ibmathrevision.com/hlai/differentiation-exam-practice
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