Confidence Intervals and Hypothesis Testing for Means: exam practice

10 questions · 44 marks · worked solutions and markschemes

Name _____ Date _____ Score / 44

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/confidence-intervals-and-hypothesis-testing-for-means-exam-practice

1. **2 marks · Paper 2 style · calculator allowed**

A 90% confidence interval for a mean is $[42.6,\ 47.8]$.

Find the sample mean and the margin of error. **[2]**

2. **3 marks · Paper 2 style · calculator allowed**

The masses (g) of eight biscuits are $12.1,\ 11.8,\ 12.4,\ 12.0,\ 11.6,\ 12.3,\ 12.2,\ 11.9$.

Find a 95% confidence interval for the mean mass. **[3]**

3. **3 marks · Paper 2 style · calculator allowed**

The lifetimes of 40 bulbs have $\bar{x} = 1240$ hours. The population standard deviation is 85 hours.

Find a 95% confidence interval for the mean lifetime. **[3]**

4. **5 marks · Paper 2 style · calculator allowed**

A machine should fill bags with a mean of 500 g. A sample of 10 bags has $\bar{x} = 496.2$ g and $s_{n-1} = 5.1$ g. Test at the 5% level whether the mean is less than 500 g.

Carry out the test. **[5]**

5. **5 marks · Paper 2 style · calculator allowed**

A teacher claims the mean reading time is 30 minutes. A sample of 12 students has $\bar{x} = 33.4$ and $s_{n-1} = 6.2$. Test at the 10% level whether the mean differs from 30.

Carry out the test. **[5]**

6. **7 marks · Paper 1 style · calculator allowed**

A 90% confidence interval for a population mean μ was calculated using a sample of $n = 25$ observations. The population standard deviation σ was unknown, so the t-distribution was used. The calculated interval is: $120.4 < \mu < 129.6$.

Determine the exact sample standard deviation (s_{n-1}) that was calculated from the raw data to generate this interval.

7. **3 marks · Paper 2 style · calculator allowed**

The lengths of a species of fish are normally distributed with standard deviation 6 cm. A random sample of 25 fish has mean length 48.2 cm. Find a 95% confidence interval for the population mean length.

8. **4 marks · Paper 1 style · calculator allowed**

A machine fills bags with flour. The true population standard deviation of the fill weight is known to be $\sigma = 4.2$ grams. A random sample of 35 bags is taken, and the sample mean weight is $\bar{x} = 502.5$ grams. Calculate a 95% confidence interval for the true mean weight of the flour bags. Give your limits to 1 decimal place.

9. **5 marks · Paper 2 style · calculator allowed**

The masses, in grams, of a random sample of 10 apples from an orchard are $152,\ 148,\ 160,\ 155,\ 149,\ 158,\ 151,\ 147,\ 163,\ 154$.

The masses are assumed to be normally distributed.

a. Find the sample mean and the unbiased estimate of the population standard deviation. **[2]**

b. Find a 95% confidence interval for the mean mass of apples from the orchard. **[3]**

10. **7 marks · Paper 1 style · calculator allowed**

A farmer measures the daily milk yield (in litres) of 5 cows before and after changing their diet. Let $d = (\text{After} - \text{Before})$. The differences for the 5 cows are:

$$d_1 = 1.2, \quad d_2 = 0.8, \quad d_3 = x, \quad d_4 = 1.5, \quad d_5 = 0.5$$

The farmer runs a paired t-test and finds that the mean difference $\bar{d} = 1.1$.

(a) Find the missing value x. **[2 marks]**

(b) Calculate the p-value for the test $H_0 : \mu_d = 0$ vs $H_1 : \mu_d > 0$. **[5 marks]**

Answers and worked solutions

Question 1 — 2 marks

Answer: 45.2; 2.6

$\bar{x} = 45.2$ **A1**

2.6 **A1**

Question 2 — 3 marks

Answer: $[11.81,\ 12.26]\ g$

$t-$interval, $\bar{x} = 12.0375$,

$s_{n-1} = 0.2722$ **M1A1**

$[11.81,\ 12.26]$ **A1**

Question 3 — 3 marks

Answer: $[1214,\ 1266]$ hours

$z-$interval **M1**

$1240 \pm 1.96 \times \dfrac{85}{\sqrt{40}}$ **A1**

$[1214,\ 1266]$ **A1**

Question 4 — 5 marks

Answer: $t = -2.36$, $p = 0.0214$; reject H_0

$H_0 : \mu = 500$, $H_1 : \mu < 500$ **A1**

one-tailed $t-$test, $t = -2.36$ **M1A1**

$p = 0.0214$ **A1**

$p < 0.05$: reject H_0 **R1**

Question 5 — 5 marks

Answer: $t = 1.90$, $p = 0.0840$; reject H_0

$H_0 : \mu = 30$, $H_1 : \mu \neq 30$ **A1**

$t = 1.90$ **A1**

two-tailed $p = 0.0840$ **M1A1**

$p < 0.10$: reject H_0 **R1**

Question 6 — 7 marks

Answer: 13.4

Find the sample mean $\bar{x}$ (midpoint of interval): **[M1]**

$\bar{x} = \frac{120.4+129.6}{2} = 125.0$. **[A1]**

Margin of error:

$E = 129.6 - 125.0 = 4.6$. **[A1]**

Margin of error for a $t-$interval: $E = t^* \frac{s_{n-1}}{\sqrt{n}}$, with

$df = 25 - 1 = 24$. **[M1]**

For 90% confidence, the area to the left of t^* is 0.95: InvT(0.95, 24) $\implies$ $t^* = 1.71088\ldots$ **[A1]**

Set up the equation:

$4.6 = 1.71088 \times \frac{s_{n-1}}{\sqrt{25}}$ **[M1]**

$s_{n-1} = \frac{4.6 \times 5}{1.71088} = 13.4$ ($13.443\ldots$) **[A1]**

Question 7 — 3 marks

Answer: (Lower limit) 45.8; (Upper limit) 50.6

$48.2 \pm 1.96 \times \dfrac{6}{\sqrt{25}}$ (z-interval, σ known) **(M1)**

$(45.8, 50.6)$ **A1A1**

Question 8 — 4 marks

Answer: (Lower limit) 501.1; (Upper limit) 503.9

Because the population standard deviation (σ) is known, we use a $Z-$interval. **[M1]**

Using GDC: STAT $\rightarrow$ INTR $\rightarrow$ Z $\rightarrow$ 1-Sample.

Inputs: C-Level $= 0.95$, $\sigma = 4.2$, $\bar{x} = 502.5$, $n = 35$. **[M1]**

Lower Limit $= 501.108\ldots$ **[A1]**

Upper Limit $= 503.891\ldots$ **[A1]**

Interval: $(501.1, 503.9)$ to 1 d.p.

Question 9 — 5 marks

Answer: (Sample mean (g) 153.7; ($s(g)$ 5.33; (Lower limit (g) 149.9; (Upper limit (g) 157.5

a. $\bar{x} = 153.7$ **A1**

$s_{n-1} = 5.33$ **A1**

b. $t-$interval with 9 degrees of freedom (σ unknown) **(M1)**

$(149.9, 157.5)\ g$ **A1A1**

Question 10 — 7 marks

Answer: (a) 1.5; (b) 0.00254

(a) The mean difference $\bar{d} = \frac{\sum d}{n}$:

$1.1 = \frac{1.2+0.8+x+1.5+0.5}{5}$ **[M1]**

$5.5 = 4.0 + x \implies x = 1.5$. **[A1]**

(b) $H_0 : \mu_d = 0$, $H_1 : \mu_d > 0$ (right-tailed test).

Put the differences into List 1: $1.2, 0.8, 1.5, 1.5, 0.5$. **[M1]**

Run a $1-$sample $t-$test on List 1 with $\mu_0 = 0$, $\mu > \mu_0$. **[M1]**

$\bar{d} = 1.1$, $s_d = 0.4416\ldots$, $df = 4$. **[A1]**

GDC: $t = 5.57$ ($5.5700\ldots$) **[A1]**

$p-$value $= 0.00254$ ($0.0025449\ldots$) **[A1]**

Every solution is also on ibmathrevision.com/hlai/confidence-intervals-and-hypothesis-testing-for-means-exam-practice

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