Complex Numbers: exam practice

10 questions · 38 marks · worked solutions and markschemes

Name _____ Date _____ Score / 38

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/complex-numbers-exam-practice

1. **3 marks · Paper 1 style · calculator allowed**

Let $z = 4 + 3i$.

Find z^*, $|z|$ and zz^*. **[3]**

2. **3 marks · Paper 1 style · calculator allowed**

Solve $z^2 - 6z + 13 = 0$, giving your answers in the form $a + bi$.

Find z. **[3]**

3. **3 marks · Paper 1 style · calculator allowed**

Find the real numbers p and q such that $(p + qi)(2 - i) = 7 + i$.

Find p and q. **[3]**

4. **4 marks · Paper 2 style · calculator allowed**

Two sound waves are modelled by $4\sin(t)$ and $3\sin(t+1.2)$. They are written as the imaginary parts of $4e^{it}$ and $3e^{i(t+1.2)}$.

By adding $4+3e^{1.2i}$, write their sum in the form $A\sin(t+\phi)$. **[4]**

5. **4 marks · Paper 1 style · calculator allowed**

Let $z=1-i$.

Write z in exponential form and hence find z^8. **[4]**

6. **4 marks · Paper 2 style · calculator allowed**

In an AC circuit the total impedance of two components in series is $Z=Z_1+Z_2$, where $Z_1=3+4i$ ohms and $Z_2=5e^{-0.5i}$ ohms.

Find $|Z|$ and $\arg Z$. **[4]**

7. **4 marks · Paper 1 style · calculator allowed**

Let $z_1 = 3 - 2i$ and $z_2 = 1 + 4i$.

(a) Find $z_1 z_2$. **[2]**

(b) Find $\dfrac{z_1}{z_2}$ in the form $a + bi$. **[2]**

8. **4 marks · Paper 2 style · calculator allowed**

An AC voltage is written $V = 10e^{0.6i}$ and a current $I_c = 2e^{-0.3i}$.

(a) Find the impedance $Z = \dfrac{V}{I_c}$ in exponential form. **[2]**

(b) Write Z in Cartesian form. **[2]**

9. **4 marks · Paper 1 style · calculator allowed**

Let $z = 6\left(\cos\dfrac{\pi}{4} + i\sin\dfrac{\pi}{4}\right)$ and $w = 3\left(\cos\dfrac{\pi}{12} + i\sin\dfrac{\pi}{12}\right)$.

(a) Find zw in polar form. **[2]**

(b) Write zw in the form $a + bi$. **[2]**

10. **5 marks · Paper 1 style · calculator allowed**

Let $w = -2 + 2\sqrt{3}\,i$.

(a) Find the modulus and argument of w. **[2]**

(b) Write w in exponential form, and find w^3. **[3]**

Answers and worked solutions

Question 1

3 marks

Answer: $4-3i$, 5, 25

$z^* = 4-3i$ A1

$|z| = 5$ A1

$zz^* = 25$ A1

Question 2

3 marks

Answer: $z = 3 \pm 2i$

$z = \dfrac{6 \pm \sqrt{36-52}}{2}$ M1

$\sqrt{-16} = 4i$ A1

$z = 3 \pm 2i$ A1

Question 3

3 marks

Answer: $p = \dfrac{13}{5}$, $q = \dfrac{9}{5}$

$p + qi = \dfrac{7+i}{2-i}$ M1

$= \dfrac{(7+i)(2+i)}{5} = \dfrac{13+9i}{5}$

A1

$p = \dfrac{13}{5}$, $q = \dfrac{9}{5}$ A1

Question 4

4 marks

Answer: $5.80\sin(t+0.503)$

$4 + 3\cos 1.2 + 3i\sin 1.2 = 5.087 + 2.796i$

$M1A1$

$A = 5.80$ A1

$\phi = 0.503$ A1

Question 5

4 marks

Answer: 16

$|z| = \sqrt{2}$ A1

$\arg z = -\dfrac{\pi}{4}$ A1

$z^8 = 16e^{-2\pi i}$ M1

16 A1

Question 6

4 marks

Answer: $|Z| = 7.56$ ohms, $\arg Z = 0.214$

$Z_2 = 4.388 - 2.397i$ A1

$Z = 7.388 + 1.603i$ A1

$|Z| = 7.56$ A1

$\arg Z = 0.214$ A1

Question 7

4 marks

Answer: (a) $11+10i$; (b) $-\dfrac{5}{17} - \dfrac{14}{17}i$

(a)

$3 + 12i - 2i + 8$ M1

$11 + 10i$ A1

(b)

$\dfrac{(3-2i)(1-4i)}{17}$ M1

$-\dfrac{5}{17} - \dfrac{14}{17}i$ A1

Question 8

4 marks

Answer: (a) $5e^{0.9i}$; (b) $3.11 + 3.92i$

(a)

$\dfrac{10}{2}e^{(0.6+0.3)i}$ M1

$5e^{0.9i}$ A1

(b)

$5\cos 0.9 + 5i\sin 0.9$ M1

$3.11 + 3.92i$ A1

Question 9

4 marks

Answer: (a) $18\,\text{cis}\,\dfrac{\pi}{3}$; (b) $9 + 9\sqrt{3}\,i$

(a)

$18\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$ A1A1

(b)

$18\left(\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i\right)$ M1

$9 + 9\sqrt{3}\,i$ A1

Question 10

5 marks

Answer: (a) 4, $\dfrac{2\pi}{3}$; (b) 64

(a)

$|w| = 4$ A1

$\arg w = \dfrac{2\pi}{3}$ A1

(b)

$w = 4e^{\frac{2\pi}{3}i}$ A1

$w^3 = 64e^{2\pi i}$ M1

64 A1

Every solution is also on ibmathrevision.com/hlai/complex-numbers-exam-practice
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