Complex Numbers and Argand Diagrams: exam practice

10 questions · 33 marks · worked solutions and markschemes

Name _____ Date _____ Score / 33

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/complex-numbers-and-argand-diagrams-exam-practice

1. **2 marks · Paper 1 style · calculator allowed**

Given the complex number $z = 3 - 4i$, find its modulus.

2. **2 marks · Paper 1 style · calculator allowed**

The complex numbers $z_1 = 5 + 2i$ and $z_2 = 1 - 3i$ are drawn on an Argand diagram.

Find the distance between the points representing z_1 and z_2. **[2]**

3. **3 marks · Paper 2 style · calculator allowed**

Let $z = 2e^{\frac{\pi}{6}i}$.

Find z^2, z^3 and z^4 in exponential form, and the argument of z^4 in degrees. **[3]**

4. **3 marks · Paper 1 style · calculator allowed**

The point P represents $z = a + bi$ on an Argand diagram. P is reflected in the real axis to Q.

Write down the complex number Q represents and find $z + z^*$ and $z - z^*$. **[3]**

5. **3 marks · Paper 1 style · calculator allowed**

The vertices of a triangle on an Argand diagram are 0, z and $z + w$, where $z = 3 + i$ and $w = -1 + 2i$.

Find the area of the triangle. **[3]**

6. **3 marks · Paper 1 style · calculator allowed**

Multiplying a complex number by $u = 2e^{\frac{\pi}{3}i}$ maps the point $A(1, 0)$ to B and then B to C.

Find the complex number represented by C in Cartesian form. **[3]**

7. **4 marks · Paper 1 style · calculator allowed**

Find the exact Cartesian form of z^4, given the complex number $z = \sqrt{2} - i\sqrt{2}$.

8. **4 marks · Paper 2 style · calculator allowed**

Let $w = \sqrt{3} + i$.

Find the smallest positive integer n for which w^n is real and negative, and find w^n. **[4]**

9. **7 marks · Paper 1 style · calculator allowed**

In an AC circuit, the total impedance Z_T of two components connected in parallel is given by the formula:

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Given that $Z_1 = 40 + 20i$ ohms and $Z_2 = 30 - 10i$ ohms, calculate Z_T.

Give your final answer for Z_T as an exact real number.

10. **2 marks · Paper 1 style · calculator allowed**

On an Argand diagram, the points A and B represent $z_1 = 2 + i$ and $z_2 = iz_1$.

(a) Find z_2. **[1]**

(b) Describe the transformation that maps A to B. **[1]**

Answers and worked solutions

Question 1 2 marks

Answer: 5

Use the modulus formula:

$|z| = \sqrt{3^2 + (-4)^2}$ M1

$|z| = \sqrt{9 + 16} = \sqrt{25} = 5$ A1

Question 2 2 marks

Answer: $\sqrt{41}$

$|z_1 - z_2| = |4 + 5i|$ M1

$\sqrt{41}$ A1

Question 3 3 marks

Answer: $16e^{\frac{2\pi}{3}i}$, argument $120°$

$4e^{\frac{\pi}{3}i}$, $8e^{\frac{\pi}{2}i}$ A1

$16e^{\frac{2\pi}{3}i}$ A1

$120°$ A1

Question 4 3 marks

Answer: $a - bi$; $2a$; $2bi$

$a - bi$ A1

$z + z^* = 2a$ A1

$z - z^* = 2bi$ A1

Question 5 3 marks

Answer: $\frac{7}{2}$

vertices $(0,0)$, $(3,1)$, $(2,3)$ A1

$\frac{1}{2}|3(3) - 1(2)|$ M1

$\frac{7}{2}$ A1

Question 6 3 marks

Answer: $-2 + 2\sqrt{3}\,i$

$C = u^2 = 4e^{\frac{2\pi}{3}i}$ M1A1

$-2 + 2\sqrt{3}\,i$ A1

Question 7 4 marks

Answer: -16

Find the modulus:

$r = \sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2} = \sqrt{4} = 2$ A1

Find the argument (fourth quadrant):

$\tan\theta = -1 \implies \theta = -\frac{\pi}{4}$ A1

Apply De Moivre's theorem:

$z^4 = 2^4\left(\cos\left(4 \times -\frac{\pi}{4}\right) + i\sin\left(4 \times -\frac{\pi}{4}\right)\right) = 16(\cos(-\pi) + i\sin(-\pi))$ M1

Evaluate:

$z^4 = 16(-1 + 0i) = -16$ A1

Question 8 4 marks

Answer: $n = 6$, $w^6 = -64$

$w = 2e^{\frac{\pi}{6}i}$ A1

$\frac{n\pi}{6} = \pi$ M1

$n = 6$ A1

$w^6 = -64$ A1

Question 9 7 marks

Answer: 20

Step 1: Expand the numerator

$Z_1 Z_2 = (40 + 20i)(30 - 10i)$

:

$= 1200 - 400i + 600i - 200i^2$ [M1]

$= 1200 + 200i + 200 = 1400 + 200i$ [A1]

Step 2: Add for the denominator:

$Z_1 + Z_2 = (40 + 20i) + (30 - 10i) = 70 + 10i$ [A1]

Step 3: Divide:

$Z_T = \frac{1400+200i}{70+10i} = \frac{140+20i}{7+i}$ (cancelling 10).

Multiply by the conjugate of the denominator: $\frac{(140+20i)(7-i)}{(7+i)(7-i)}$ [M1]

Numerator:

$980 - 140i + 140i - 20i^2 = 1000$ [A1]

Denominator: $49 - i^2 = 50$ [A1]

$Z_T = \frac{1000}{50} = 20$ ohms [A1]

Question 10 2 marks

Answer: (a) $-1 + 2i$

(a)

$-1 + 2i$ A1

(b)

rotation of $\frac{\pi}{2}$ anticlockwise about the origin A1

Every solution is also on ibmathrevision.com/hlai/complex-numbers-and-argand-diagrams-exam-practice
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