Chi-Square Goodness of Fit, Independence and Error Types: exam practice

10 questions · 43 marks · worked solutions and markschemes

Name _____ Date _____ Score / 43

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/chi-square-goodness-of-fit-independence-and-error-types-exam-practice

1. **2 marks · Paper 1 style · calculator allowed**

A χ^2 goodness of fit test is performed to see if data fits a Poisson distribution. The data is grouped into 5 categories: 0, 1, 2, 3, and ≥ 4 events. The parameter λ is estimated from the sample data. Calculate the exact number of degrees of freedom for this test.

2. **3 marks · Paper 1 style · calculator allowed**

The mean of a normal population with standard deviation $\sigma = 5$ is tested with a sample of size $n = 25$: $H_0 : \mu = 50$ is rejected in favour of $H_1 : \mu < 50$ if $\bar{x} < 48.355$. Given that the true population mean is actually $\mu = 48$, calculate the probability of making a Type II error, correct to 3 significant figures.

3. **4 marks · Paper 1 style · calculator allowed**

A researcher conducts a Chi-Square test for independence to see if eye color (Blue, Green, Brown, Hazel) is independent of hair color (Blonde, Brown, Black, Red, Other).

Calculate the exact number of degrees of freedom (df) for this test.

4. **2 marks · Paper 1 style · calculator allowed**

A test is carried out at the 5% significance level.

Define a type I error and state its probability. **[2]**

5. **2 marks · Paper 1 style · calculator allowed**

A medical test has H_0: the patient is healthy.

Describe a type II error in context. **[2]**

6. **5 marks · Paper 2 style · calculator allowed**

A dice is rolled 120 times. The frequencies of 1 to 6 are $15,\ 22,\ 18,\ 25,\ 17,\ 23$. Test at the 5% level whether the dice is fair.

Carry out the test. **[5]**

7. **5 marks · Paper 2 style · calculator allowed**

The table shows preferred sport by gender for 200 students. Test at the 5% level whether preference is independent of gender.

	Football	Tennis	Swimming
Male	45	25	30
Female	30	35	35

Carry out the test. **[5]**

8. **5 marks · Paper 2 style · calculator allowed**

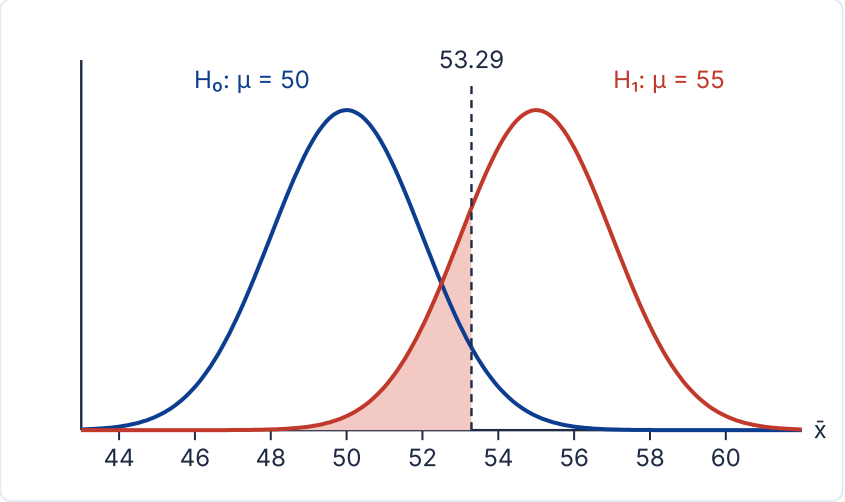
A seed company claims its mix is 50% red, 30% white and 20% yellow. In a sample of 150 flowers there are 64 red, 52 white and 34 yellow.

Test the claim at the 5% level. **[5]**

9. **7 marks · Paper 1 style · calculator allowed**

The diagram shows the distributions for a 1-tailed hypothesis test. $H_0 : \mu = 50$, against $H_1 : \mu = 55$. The population standard deviation is $\sigma = 10$, and sample size $n = 25$. The decision rule is to reject H_0 if the sample mean $\bar{x} > 53.29$.

Calculate the exact probability of making a **Type II error** (β).



10. **8 marks · Paper 1 style · calculator allowed**

A researcher claims that the heights of plants in a greenhouse follow a Normal distribution with $\mu = 120$ cm and $\sigma = 15$ cm. He categorizes 200 plants into three intervals: - Group 1: < 105 cm - Group 2: 105 cm to 135 cm - Group 3: > 135 cm

Calculate the expected frequency (f_e) for each of the three groups, correct to 1 decimal place.

Answers and worked solutions

Question 1

2 marks

Answer: 3

Use $\nu = k - 1 - m$ with $k = 5$ categories and $m = 1$ estimated parameter (λ): $\nu = 5 - 1 - 1$ M1

$\nu = 3$ A1

Question 2

3 marks

Answer: 0.361

A Type II error is failing to reject H_0 when it is false, so find $P(\bar{X} \geq 48.355)$ where $\bar{X} \sim N(48, 1^2)$ M1

Standardise: $z = \frac{48.355 - 48}{1} = 0.355$ (A1)

$P(Z \geq 0.355) = 0.361$ (3 s.f.) A1

Question 3

4 marks

Answer: 12

Formula for degrees of freedom in an $r \times c$ contingency table: $df = (r-1)(c-1)$. [M1]

Eye color has 4 categories (rows $r = 4$) and hair color has 5 categories (columns $c = 5$). [A1]

Substitute: $df = (4-1)(5-1)$. [M1]

$df = 3 \times 4 = 12$. [A1]

Question 4

2 marks

Answer: 0.05

rejecting H_0 when it is true A1

0.05 A1

Question 5

2 marks

Answer: a proof: the worked solution above is the full answer

not rejecting H_0 when it is false A1

concluding a sick patient is healthy A1

Question 6

5 marks

Answer: $\chi^2 = 3.8$, $p = 0.579$; do not reject H_0

H_0: the dice is fair; H_1: it is not A1

expected 20 each A1

$\chi^2 = 3.8$, $p = 0.579$ A1A1

$p > 0.05$: do not reject H_0; no evidence the dice is unfair R1

Question 7

5 marks

Answer: $\chi^2 = 5.05$, $p = 0.0800$; do not reject H_0

H_0: independent A1

$\text{df} = 2$ A1

$\chi^2 = 5.05$ A1

$p = 0.0800$ A1

$p > 0.05$: do not reject H_0 R1

Question 8

5 marks

Answer: $\chi^2 = 3.24$, $p = 0.198$; do not reject

expected $75,\ 45,\ 30$ A1

$\text{df} = 2$ A1

$\chi^2 = 3.24$ A1

$p = 0.198$ A1

do not reject the claim R1

Question 9

7 marks

Answer: 0.196

A Type II error (β) is the probability of **failing to reject H_0 when H_1 is true**. [M1]

The decision rule is to accept H_0 if $\bar{x} \leq 53.29$. [M1]

We must calculate $P(\bar{x} \leq 53.29)$ using the H_1 distribution ($\mu = 55$). [M1]

Since we are dealing with a sample mean $\bar{x}$, the standard deviation of the sampling distribution is $\frac{\sigma}{\sqrt{n}}$. [R1]

Standard error $= \frac{10}{\sqrt{25}} = \frac{10}{5} = 2$. [M1][A1]

Use GDC Normal CDF: Lower $= -1 \times 10^{99}$, Upper $= 53.29$, $\mu = 55$, $\sigma = 2$.

$\beta = P(\bar{x} \leq 53.29) = 0.196$ (to 3 s.f.). [A1]

Question 10

8 marks

Answer: (Group 1) 31.7; (Group 2) 136.5; (Group 3) 31.7

We use the Normal distribution $N(120, 15^2)$ to find the theoretical probability of falling into each group. [M1]

Group 1 (< 105): Normal CDF (Lower$= -9E99$, Upper$= 105$, $\mu = 120, \sigma = 15$) $= 0.15865...$ [M1][A1]

Group 2 ($105 - 135$): Normal CDF (Lower$= 105$, Upper$= 135$) $= 0.68268...$ [M1][A1]

Group 3 (> 135): Normal CDF (Lower$= 135$, Upper$= 9E99$) $= 0.15865...$ [A1]

To find expected frequencies, multiply probabilities by total plants ($n = 200$). [M1]

$f_e(\text{Group 1}) = 0.15865 \times 200 = 31.7$.

$f_e(\text{Group 2}) = 0.68268 \times 200 = 136.5$.

$f_e(\text{Group 3}) = 0.15865 \times 200 = 31.7$. [A1]

Every solution is also on ibmathrevision.com/hlai/chi-square-goodness-of-fit-independence-and-error-types-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.