Calculus: exam practice

10 questions · 37 marks · worked solutions and markschemes

Name _____ Date _____ Score / 37

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/calculus-exam-practice

1. **3 marks · Paper 2 style · calculator allowed**

Use Euler's method with $h = 0.2$ to estimate $y(1)$ for $\frac{dy}{dx} = x - y$, $y(0) = 2$.

Find the estimate. **[3]**

2. **3 marks · Paper 2 style · calculator allowed**

The region between $y = \sqrt{x}$, the x-axis and $x = 9$ is rotated $360°$ about the x-axis.

Find the exact volume. **[3]**

3. **3 marks · Paper 2 style · calculator allowed**

Use Euler's method with $h = 0.1$ to estimate $y(0.3)$ for $\frac{dy}{dx} = xy$, $y(0) = 1$.

Find the estimate. **[3]**

4. **3 marks · Paper 1 style · calculator allowed**

Solve $\frac{dy}{dx} = 3y$, $y(0) = 4$.

Find y. **[3]**

5. **3 marks · Paper 2 style · calculator allowed**

A system is $\dot{x} = x + 2y$, $\dot{y} = 3x + 2y$.

Find the eigenvalues of the matrix and classify the equilibrium. **[3]**

6. **4 marks · Paper 2 style · calculator allowed**

Consider the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $y(0) = 1$.

Use Euler's method with a step size of $h = 0.1$ to estimate the value of $y(0.2)$.

7. **4 marks · Paper 2 style · calculator allowed**

A cup of tea is left to cool in a room. Its temperature, T °C, after t minutes satisfies the differential equation $\frac{dT}{dt} = -0.1(T - 20)$, with $T(0) = 80$.

Use Euler's method with a step size of $h = 1$ minute to estimate the temperature of the tea after 2 minutes.

8. 4 marks · Paper 2 style · calculator allowed

The motion of a mass on a spring satisfies $\frac{d^2x}{dt^2} + 4x = 0$. Write it as a system with $y = \frac{dx}{dt}$.

Write the system and classify the equilibrium point at the origin. **[4]**

9. 4 marks · Paper 2 style · calculator allowed

The numbers of rabbits, x, and foxes, y, in a nature reserve are modelled by the coupled differential equations

$\frac{dx}{dt} = 0.4x - 0.01xy, \quad \frac{dy}{dt} = -0.3y + 0.005xy,$

where t is measured in years. Initially $x = 50$ and $y = 20$.

Use Euler's method with a step size of $h = 0.5$ to estimate the numbers of rabbits and foxes when $t = 1$. Give your answers correct to 3 significant figures.

10. 6 marks · Paper 2 style · calculator allowed

A bowl is modelled by rotating the curve $y = 0.25x^2$, for $0 \le y \le 4$, through 2π radians about the y-axis. All lengths are in centimetres.

1. Find the volume of the bowl.
2. Water is poured into the bowl to a depth of h cm. Find the depth of water when the bowl is exactly half full.

Answers and worked solutions

Question 1

3 marks

Answer: 0.983

$y_{n+1} = y_n + 0.2(x_n - y_n)$ **M1**

five steps **M1**

0.983 **A1**

Question 2

3 marks

Answer: $\dfrac{81\pi}{2}$

$\pi \displaystyle\int_0^9 x\,dx$ *M*1A1

$\dfrac{81\pi}{2}$ **A1**

Question 3

3 marks

Answer: 1.0302

$y_1 = 1$ **A1**

$y_2 = 1.01$ **A1**

$y_3 = 1.0302$ **A1**

Question 4

3 marks

Answer: $y = 4e^{3x}$

$\displaystyle\int \frac{dy}{y} = \int 3\,dx$ **M1**

$\ln y = 3x + c$ **A1**

$y = 4e^{3x}$ **A1**

Question 5

3 marks

Answer: $\lambda = 4, -1$; a saddle

$\lambda^2 - 3\lambda - 4 = 0$ **M1**

$\lambda = 4,\ -1$ **A1**

saddle point **A1**

Question 6

4 marks

Answer: 1.22

$x_0 = 0, y_0 = 1$.

$y_1 = y_0 + h(x_0 + y_0) = 1 + 0.1(0 + 1)$ **M1**

$y_1 = 1 + 0.1 = 1.1$. (So $x_1 = 0.1, y_1 = 1.1$). **A1**

$y_2 = y_1 + h(x_1 + y_1) = 1.1 + 0.1(0.1 + 1.1)$ **M1**

$y_2 = 1.1 + 0.1(1.2) = 1.1 + 0.12 = 1.22$.

$y(0.2) \approx 1.22$. **A1**

Question 7

4 marks

Answer: 68.6

$t_0 = 0, T_0 = 80$.

$T_1 = T_0 + h\,(-0.1(T_0 - 20)) = 80 + 1(-0.1 \times 60)$ **M1**

$T_1 = 80 - 6 = 74$. (So $t_1 = 1, T_1 = 74$). **A1**

$T_2 = T_1 + h\,(-0.1(T_1 - 20)) = 74 + 1(-0.1 \times 54)$ **M1**

$T_2 = 74 - 5.4 = 68.6$.

$T(2) \approx 68.6$ °C. **A1**

Question 8

4 marks

Answer: $\dot{x} = y$, $\dot{y} = -4x$; a centre

$\dot{x} = y$, $\dot{y} = -4x$ **A1A1**

eigenvalues $\pm 2i$ **A1**

centre (oscillation) **A1**

Question 9

4 marks

Answer: (Rabbits) 60.6; (Foxes) 19.3

$t_0 = 0, x_0 = 50, y_0 = 20$.

$x_1 = x_0 + h(0.4x_0 - 0.01x_0y_0) = 50 + 0.5(20 - 10)$

and

$y_1 = y_0 + h(-0.3y_0 + 0.005x_0y_0) = 20 + 0.5(-6 + 5)$ **M1**

$x_1 = 55,\ y_1 = 19.5$. (So $t_1 = 0.5$). **A1**

$x_2 = 55 + 0.5\,(0.4(55) - 0.01(55)(19.5)) = 55 + 0.5(22 - 10.725)$

and

$y_2 = 19.5 + 0.5\,(-0.3(19.5) + 0.005(55)(19.5)) = 19.5 + 0.5(-5.85$ **M1**

$x_2 = 60.6375,\ y_2 = 19.25625$.

At $t = 1$: $x \approx 60.6$ rabbits and $y \approx 19.3$ foxes. **A1**

Question 10

6 marks

Answer: (a) $(\text{cm}^3)\ 32\pi$; (b) (cm) $2\sqrt{2}$

1. Rearrange: $x^2 = 4y$. **[M1]**
   $V = \pi \int_0^4 x^2\,dy = \pi \int_0^4 4y\,dy$ **[M1]**
   $V = 32\pi \approx 101\ \text{cm}^3$ $(100.53...)$. **[A1]**
2. Volume to depth h:
   $V(h) = \pi \int_0^h 4y\,dy = 2\pi h^2$. **[M1]**
   Set $2\pi h^2 = 16\pi$ **[A1]**
   $h = \sqrt{8} \approx 2.83$ cm. **[A1]**

Every solution is also on ibmathrevision.com/hlai/calculus-exam-practice
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