Bivariate Data: exam practice

10 questions · 31 marks · worked solutions and markschemes

Name _____ Date _____ Score / 31

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/bivariate-data-exam-practice

1. **2 marks · Paper 1 style · calculator allowed**

A scatter diagram of height against arm span for 30 students has $r = 0.91$.

Describe the correlation and state whether arm span could be predicted reliably from height within the data range. **[2]**

2. **2 marks · Paper 1 style · calculator allowed**

A data set has $r = 0.12$ but the points lie close to a curve.

Explain what this shows about the relationship. **[2]**

3. **2 marks · Paper 1 style · calculator allowed**

Explain when Spearman's rank correlation is more suitable than Pearson's r.

Explain. **[2]**

4. **3 marks · Paper 2 style · calculator allowed**

Two judges rank six singers. Judge A's ranks are $1, 2, 3, 4, 5, 6$ and judge B's ranks of the same singers are $3, 1, 4, 2, 6, 5$.

Find Spearman's rank correlation coefficient and comment on the agreement. **[3]**

5. **3 marks · Paper 2 style · calculator allowed**

The age x (months) and mass y (kg) of six young goats are recorded.

x	12	18	25	31	40	46
y	3.2	4.1	5.5	6.0	7.6	8.3

Find the regression line of y on x and estimate the mass at 36 months. **[3]**

6. **3 marks · Paper 2 style · calculator allowed**

Seven students' scores in maths (x) and physics (y) are shown.

x	65	72	80	58	90	77	84
y	70	75	78	60	88	69	82

Find Spearman's rank correlation coefficient. **[3]**

7. **3 marks · Paper 2 style · calculator allowed**

The price x (dollars) and weekly sales y (hundreds) of a product are shown.

x	5	10	15	20	25
y	28	22	19	13	9

Find the regression line and interpret its gradient. **[3]**

8. **4 marks · Paper 2 style · calculator allowed**

A researcher fits a model $\hat{y} = 3x^2 - x$ to a dataset. The observed values y for three data points are given below.

x	Observed y	Predicted $\hat{y}$	Residual $(y - \hat{y})$
1	3	a	b
2	12	10	2
3	20	24	-4

1. Find the values of a and b.
2. Calculate the Sum of Squared Residuals (SS_{res}) for these three points.

9. **4 marks · Paper 2 style · calculator allowed**

The number of absences x and the exam score y of seven students are shown.

x	3	5	6	8	10	11	13
y	62	58	55	49	45	40	37

(a) Find the equation of the regression line of y on x. **[2]**

(b) Find r and interpret it. **[2]**

10. **5 marks · Paper 2 style · calculator allowed**

The height, h in metres, of a projectile is recorded at various horizontal distances, d in metres, from the launch point.

Distance, d	10	20	30	40	50
Height, h	18	32	42	48	50

1. Using your Graphic Display Calculator, find the equation of the quadratic regression curve $h = ad^2 + bd + c$ for this data.
2. Using your model, predict the maximum height reached by the projectile.

Answers and worked solutions

Question 1 — 2 marks

Answer: strong positive; yes

strong positive **A1**

yes, strong correlation and interpolation **R1**

Question 2 — 2 marks

Answer: a proof: the worked solution above is the full answer

no linear correlation **A1**

but a non-linear relationship may exist **R1**

Question 3 — 2 marks

Answer: a proof: the worked solution above is the full answer

when the relationship is monotonic but not linear **A1**

or the data are ranks / ordinal **A1**

Question 4 — 3 marks

Answer: $r_s = 0.657$; moderate agreement

GDC on ranks **(M1)**

$r_s = 0.657$ **A1**

moderate positive agreement **A1**

Question 5 — 3 marks

Answer: 6.89 kg, from $y = 0.151x + 1.45$

$y = 0.151x + 1.45$ **A1A1**

6.89 kg **A1**

Question 6 — 3 marks

Answer: $r_s = 0.893$

rank both sets **M1**

GDC **(M1)**

$r_s = 0.893$ **A1**

Question 7 — 3 marks

Answer: $y = -0.94x + 32.3$

$y = -0.94x + 32.3$ **A1A1**

each extra dollar cuts sales by about 94 **A1**

Question 8 — 4 marks

Answer: (a) 2; (b) 1; (SS_{res}) 21

1. Substitute $x = 1$ into the model: $a = 3(1)^2 - 1 = 2$. **A1**
   Residual = observed − predicted: $b = 3 - 2 = 1$. **A1**
2. Square and add the residuals: $SS_{res} = 1^2 + 2^2 + (-4)^2$. **M1**
   $SS_{res} = 1 + 4 + 16 = 21$. **A1**

Question 9 — 4 marks

Answer: (a) $y = -2.62x + 70.4$; (b) $r = -0.996$

(a)

GDC **(M1)**

$y = -2.62x + 70.4$ **A1**

(b)

$r = -0.996$ **A1**

strong negative linear correlation **A1**

Question 10 — 5 marks

Answer: (a) -0.02; (b) 2; (c) 0; (Maximum height (m) 50

1. Enter the data into the GDC and use quadratic regression. **(M1)**
   $a = -0.02$, $b = 2$ **A1**
   $c = 0$ **A1**
   Model: $h = -0.02d^2 + 2d$.
2. The maximum is at the vertex: $d = \frac{-b}{2a} = \frac{-2}{2(-0.02)} = 50$. **M1**
   Substitute $d = 50$: $h = -0.02(50^2) + 2(50) = -50 + 100 = 50$ m. **A1**
   (Alternatively, the maximum may be found directly from the GDC graph.)

Every solution is also on ibmathrevision.com/hlai/bivariate-data-exam-practice
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