Advanced Modelling: exam practice

10 questions · 42 marks · worked solutions and markschemes

Name _____ Date _____ Score / 42

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/advanced-modelling-exam-practice

1. **3 marks · Paper 2 style · calculator allowed**

A cubic model $h(x) = ax^3 + bx^2 + cx$ for the height of a ramp passes through $(1, 4)$, $(2, 10)$ and $(3, 12)$.

Find a, b and c. **[3]**

2. **3 marks · Paper 2 style · calculator allowed**

The height of a tide is $h(t) = 2.4\sin(0.5t) + 3.1$ metres, t hours after midnight.

Find the first two times after midnight when the height is 4 m. **[3]**

3. **5 marks · Paper 1 style · calculator allowed**

For the logistic model $P(t) = \frac{5000}{1+24e^{-0.3t}}$, find the initial population at $t = 0$.

4. **6 marks · Paper 1 style · calculator allowed**

For the tide model $D(t) = 4\sin\left(\frac{\pi}{6}t\right) + 6$, find the first time $t > 0$ when the depth is exactly 8 metres.

5. **4 marks · Paper 2 style · calculator allowed**

A logistic model $P(t) = \dfrac{L}{1 + Ae^{-kt}}$ has $P(0) = 50$, $L = 800$ and $P(4) = 300$.

Find A and k. **[4]**

6. **2 marks · Paper 2 style · calculator allowed**

The number of people who have heard a rumour t hours after it starts is modelled by $P(t) = \dfrac{500}{1 + 24e^{-0.4t}}$.

a. Find the number of people who had heard the rumour at $t = 0$. **[1]**

b. Write down the long-term number of people who will hear it. **[1]**

7. **4 marks · Paper 1 style · calculator allowed**

A radioactive substance follows the decay model $M(t) = 200e^{-0.1t}$. Find the initial mass and the mass remaining after exactly 10 years.

8. **4 marks · Paper 2 style · calculator allowed**

A cubic model $y = ax^3 + bx^2 + cx + d$ passes through the points $(0, 0)$, $(1, 4)$, $(2, 6)$ and $(3, 9)$. Use technology to find a, b, c and d.

9. **4 marks · Paper 2 style · calculator allowed**

The number of people who have heard a rumour, t hours after it starts, is modelled by $N(t) = \dfrac{2000}{1 + 49e^{-0.6t}}$.

(a) Find the number who have heard the rumour at the start. **[1]**

(b) Find the time when 1000 people have heard it. **[2]**

(c) Write down the limiting value of N. **[1]**

10. **7 marks · Paper 1 style · calculator allowed**

The population of a species of insects in a closed environment is modelled by the logistic equation:

$$P(t) = \frac{L}{1 + Ce^{-kt}}$$

Initially, the population is 20. After 5 days, the population is 80. The environment can sustain a maximum of 500 insects.

Set up the necessary equations and algebraically solve for the exact values of L, C, and k.

Answers and worked solutions

Question 1 — 3 marks

Answer: $a = -1$, $b = 4$, $c = 1$

$a + b + c = 4$,
$8a + 4b + 2c = 10$,
$27a + 9b + 3c = 12$ $M1A1$

$a = -1$, $b = 4$, $c = 1$ **A1**

Question 2 — 3 marks

Answer: $t = 0.769$ and $t = 5.51$ hours

$\sin(0.5t) = 0.375$ **M1**

$t = 0.769$ **A1**

$t = 5.51$ **A1**

Question 3 — 5 marks

Answer: 200

Substitute $t = 0$:
$P(0) = \frac{5000}{1+24e^0}$. **M1**
Use $e^0 = 1$: denominator $= 1 + 24$. **A1**
$= 25$. **A1**
Divide: $P(0) = \frac{5000}{25}$. **M1**
$P(0) = 200$. **A1**

Question 4 — 6 marks

Answer: 1

Set $D(t) = 8$:
$4\sin\left(\frac{\pi}{6}t\right) + 6 = 8$. **M1**
Rearrange: $4\sin\left(\frac{\pi}{6}t\right) = 2$. **A1**
$\sin\left(\frac{\pi}{6}t\right) = 0.5$. **A1**
The first positive angle with sine 0.5 is $\frac{\pi}{6}$ (the next is $\frac{5\pi}{6}$, giving a later time $t = 5$). **M1**
$\frac{\pi}{6}t = \frac{\pi}{6}$. **A1**
$t = 1$. **A1**

Question 5 — 4 marks

Answer: $A = 15$, $k = 0.549$

$50 = \frac{800}{1+A}$, $A = 15$ $M1A1$
$300 = \frac{800}{1+15e^{-4k}}$ **M1**
$k = 0.549$ **A1**

Question 6 — 2 marks

Answer: (a) 20; (b) 500

a. $\frac{500}{25} = 20$ **A1**
b. 500 **A1**

Question 7 — 4 marks

Answer: (Initial mass) 200; (Mass after 10 years) 73.6

Initial mass occurs at $t = 0$:
$M(0) = 200e^0$. **M1**
$M(0) = 200$. **A1**
Substitute $t = 10$:
$M(10) = 200e^{-0.1(10)} = 200e^{-1}$. **M1**
$M(10) \approx 73.6$ (3 s.f.). **A1**

Question 8 — 4 marks

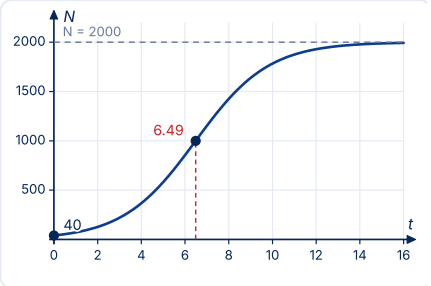
Answer: $a = 0.5$; $b = -2.5$; $c = 6$; $d = 0$

$d = 0$, $a + b + c = 4$,
$8a + 4b + 2c = 6$,
$27a + 9b + 3c = 9$ **(M1)(A1)**
$a = 0.5,\ b = -2.5,\ c = 6,\ d = 0$ **A2**

Question 9 — 4 marks

Answer: (a) 40; (b) 6.49 hours; (c) 2000

(a)
$\frac{2000}{50} = 40$ **A1**
(b)
$49e^{-0.6t} = 1$ **M1**
$t = 6.49$ hours **A1**
(c)
2000 **A1**



Starts at 40, half-way at $t = 6.49$, limit 2000

Question 10 — 7 marks

Answer: (L) 500; (C) 24; (k) $-\ln(7/32)/5$

Maximum capacity gives $L = 500$. **[A1]**
At $t = 0$:
$20 = \frac{500}{1+Ce^0} \implies 1 + C = \frac{500}{20} = 25 \implies C = 24$. **[M1][A1]**
At $t = 5$:
$80 = \frac{500}{1+24e^{-5k}} \implies 1 + 24e^{-5k} = \frac{500}{80} = 6.25$. **[M1]**
$24e^{-5k} = 5.25 \implies e^{-5k} = \frac{5.25}{24} = \frac{7}{32}$. **[M1]**
$k = \frac{\ln(7/32)}{-5} = -\frac{1}{5}\ln\left(\frac{7}{32}\right)$. **[A1][A1]**

Every solution is also on ibmathrevision.com/hlai/advanced-modelling-exam-practice
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