Advanced Integration and Kinematics: exam practice

10 questions · 41 marks · worked solutions and markschemes

Name _____ Date _____ Score / 41

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/advanced-integration-and-kinematics-exam-practice

1. **4 marks · Paper 1 style · calculator allowed**

Find the indefinite integral:

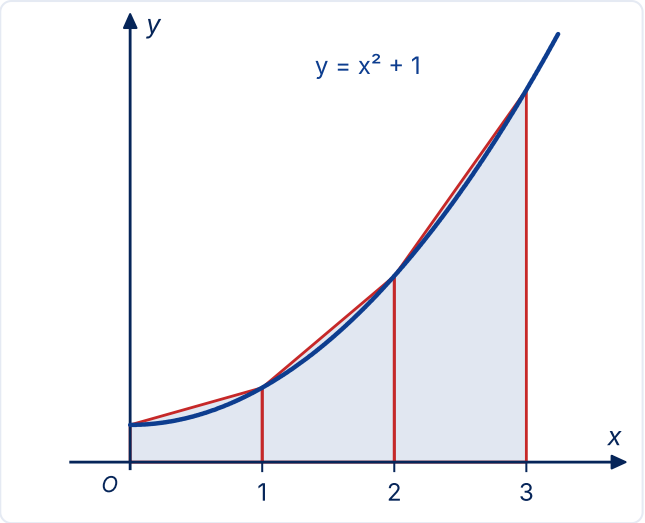
$$\int (4x^3 - 6x^2 + 5)\,dx$$

2. **4 marks · Paper 1 style · calculator allowed**

Find the exact indefinite integral:

$$\int \left(e^{3x+1} + \frac{2}{x}\right) dx$$

3. **5 marks · Paper 1 style · calculator allowed**



Use the trapezoidal rule with exactly $n = 3$ intervals to approximate the area under the curve $y = x^2 + 1$ from $x = 0$ to $x = 3$.

4. **3 marks · Paper 2 style · calculator allowed**

The region under the curve $y = e^{0.5x}$, between $x = 0$ and $x = 2$, is rotated through 2π about the x-axis. Find the volume of the solid formed.

5. **3 marks · Paper 2 style · calculator allowed**

The region under $y = e^{0.5x}$ for $0 \leq x \leq 2$ is rotated $360°$ about the x-axis.

Find the volume. **[3]**

6. **3 marks · Paper 1 style · calculator allowed**

Find $\int x(x^2+3)^3\,dx$ using a substitution.

Find the integral. **[3]**

7. **4 marks · Paper 2 style · calculator allowed**

A particle has velocity $v(t) = 3t^2 - 18t + 24$ m/s.

Find the total distance travelled in the first 5 seconds. **[4]**

8. **5 marks · Paper 1 style · calculator allowed**

Find the exact area of the region totally bounded by the curve $y = 4x - x^2$ and the x-axis.

9. **6 marks · Paper 1 style · calculator allowed**

A particle moves in a straight line with velocity $v(t) = t^2 - 3t + 2 \text{ ms}^{-1}$ for $t \geq 0$.

Calculate the exact total distance travelled by the particle in the first 4 seconds of motion.

10. **4 marks · Paper 2 style · calculator allowed**

A particle has acceleration $a(t) = 6t - 4$ m/s^2, $v(0) = 2$ m/s and $s(0) = 0$.

Find $s(t)$ and the displacement at $t = 3$. **[4]**

Answers and worked solutions

Question 1 — 4 marks

Answer: $x^4 - 2x^3 + 5x + C$

Use the power rule for integration:

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$. M1

$= \frac{4x^4}{4} - \frac{6x^3}{3} + 5x + C$. M1 A1

$= x^4 - 2x^3 + 5x + C$. A1

(Note: do not forget the constant of integration $+C$.)

Question 2 — 4 marks

Answer:

$e^(3x + 1)/3 + 2ln|x| + C$

Separate the integral into two standard forms. M1

The antiderivative of e^{kx+c} is $\frac{1}{k}e^{kx+c}$. M1

The antiderivative of $\frac{1}{x}$ is $\ln|x|$. M1

$= \frac{1}{3}e^{3x+1} + 2\ln|x| + C$. A1

Question 3 — 5 marks

Answer: 12.5

Calculate the width of each interval:

$h = \frac{b-a}{n} = \frac{3-0}{3} = 1$. M1

Find the y—values at $x = 0, 1, 2, 3$:

$y_0 = (0)^2 + 1 = 1$.

$y_1 = (1)^2 + 1 = 2$.

$y_2 = (2)^2 + 1 = 5$.

$y_3 = (3)^2 + 1 = 10$. A1 A1

Trapezoidal Rule formula:

$A \approx \frac{1}{2}h[(y_0 + y_n) + 2(y_1 + y_2 + \ldots)]$. M1

$A \approx \frac{1}{2}(1)[(1 + 10) + 2(2 + 5)]$.

$A \approx 0.5[11 + 14] = 0.5[25] = 12.5$. A1

Question 4 — 3 marks

Answer: $\pi(e^2 - 1)$

$V = \pi\int_0^2 \left(e^{0.5x}\right)^2 dx = \pi\int_0^2 e^x\,dx$ $M1$A1

$= \pi(e^2 - 1) = 20.1$ A1

Question 5 — 3 marks

Answer: 20.1

$\pi\int_0^2 e^x\,dx$ $M1$A1

20.1 A1

Question 6 — 3 marks

Answer: $\frac{(x^2+3)^4}{8} + c$

$u = x^2 + 3$ M1

$\frac{1}{2}\int u^3\,du$ A1

$\frac{(x^2+3)^4}{8} + c$ A1

Question 7 — 4 marks

Answer: $28\ m$

$v = 0$ at $t = 2, 4$ A1

$\int_0^5 |v|\,dt$ M1

$20 + 4 + 4$ A1

$28\ m$ A1

Question 8 — 5 marks

Answer: $32/3$

First, find where the curve crosses the x—axis by setting $y = 0$: M1

$4x - x^2 = 0 \implies x(4 - x) = 0 \implies x = 0$ and $x = 4$. A1

Set up the definite integral for area:

$A = \int_0^4 (4x - x^2)\,dx$. M1

Antiderivative: $[2x^2 - \frac{x^3}{3}]_0^4$. A1

Substitute bounds:

$\left(2(4)^2 - \frac{4^3}{3}\right) - (0) = 32 - \frac{64}{3}$.

Area $= \frac{96}{3} - \frac{64}{3} = \frac{32}{3}$. A1

Question 9 — 6 marks

Answer: $17/3$

Total distance requires finding when the particle changes direction ($v(t) = 0$). M1

$t^2 - 3t + 2 = 0 \implies (t-1)(t-2) = 0 \implies t = 1$ and $t = 2$. A1

Distance $= \int_0^4 |t^2 - 3t + 2|dt$. M1

We must split the integral:

$\int_0^1 v(t)dt - \int_1^2 v(t)dt + \int_2^4 v(t)dt$. M1

$s(t) = \int v(t)dt = \frac{t^3}{3} - \frac{3t^2}{2} + 2t$.

$s(0) = 0$, $s(1) = \frac{5}{6}$, $s(2) = \frac{2}{3}$, $s(4) = \frac{16}{3}$. A1

Distance

$= |s(1) - s(0)| + |s(2) - s(1)| + |s(4) - s(2)|$.

$= |\frac{5}{6} - 0| + |\frac{2}{3} - \frac{5}{6}| + |\frac{16}{3} - \frac{2}{3}| = \frac{5}{6} + \frac{1}{6} + \frac{14}{3}$.

$= 1 + \frac{14}{3} = \frac{17}{3}$ (or $5\frac{2}{3}\ m$). A1

Question 10 — 4 marks

Answer: $s = t^3 - 2t^2 + 2t$; $15\ m$

$v = 3t^2 - 4t + 2$ A1

$s = t^3 - 2t^2 + 2t$ A1A1

$s(3) = 15\ m$ A1

Every solution is also on ibmathrevision.com/hlai/advanced-integration-and-kinematics-exam-practice
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