Advanced Differentiation and Related Rates: exam practice

10 questions · 39 marks · worked solutions and markschemes

Name _____ Date _____ Score / 39

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlai/advanced-differentiation-and-related-rates-exam-practice

1. **2 marks · Paper 1 style · calculator allowed**

Differentiate $y = \sqrt{4x^2 + 1}$.

Find $\frac{dy}{dx}$. **[2]**

2. **3 marks · Paper 2 style · calculator allowed**

The radius of a circular oil spill grows at 0.5 m/s.

Find the rate at which the area grows when the radius is 20 m. **[3]**

3. **3 marks · Paper 2 style · calculator allowed**

Water fills a cone (vertex down) of semi-vertical angle $45°$ at $3\ \text{cm}^3/\text{s}$, so $V = \frac{1}{3}\pi h^3$.

Find the rate of change of the depth when the depth is 6 cm. **[3]**

4. **3 marks · Paper 2 style · calculator allowed**

A spherical balloon's volume increases at $100\ \text{cm}^3/\text{s}$.

Find the rate of increase of the radius when $r = 8$ cm. **[3]**

5. **4 marks · Paper 1 style · calculator allowed**

Differentiate $f(x) = x^2 e^{-x}$ and find the x-coordinates of its stationary points.

Find $f'(x)$ and the stationary points. **[4]**

6. **4 marks · Paper 1 style · calculator allowed**

Differentiate $y = \dfrac{\ln x}{x}$ and find the coordinates of its maximum.

Find $\dfrac{dy}{dx}$ and the maximum. **[4]**

7. **4 marks · Paper 1 style · calculator allowed**

The radius of a growing crystal, r mm, at time t hours is $r = (2t+1)^{\frac{1}{3}}$. Find the exact rate at which the radius is increasing when $t = 13$.

8. **4 marks · Paper 2 style · calculator allowed**

A 5 m ladder slides down a wall. The foot moves away from the wall at 0.4 m/s.

Find how fast the top slides down when the foot is 3 m from the wall. **[4]**

9. **8 marks · Paper 1 style · calculator allowed**

Find the exact x-coordinates of all stationary points of the function $f(x) = e^{\sin x}$ in the domain $0 \leq x \leq 2\pi$.

10. **4 marks · Paper 1 style · calculator allowed**

The sides of a square are increasing at $0.3\ \text{cm s}^{-1}$.

a. Find the rate at which the length of a diagonal is increasing. **[2]**

b. Find the rate at which the area is increasing when the sides are 8 cm long. **[2]**

Answers and worked solutions

Question 1 — 2 marks

Answer: $\dfrac{4x}{\sqrt{4x^2+1}}$

chain rule **M1**

$\dfrac{4x}{\sqrt{4x^2+1}}$ **A1**

Question 2 — 3 marks

Answer: $62.8\ \text{m}^2/\text{s}$

$\dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt}$ **M1**

$2\pi(20)(0.5)$ **A1**

$62.8\ \text{m}^2/\text{s}$ **A1**

Question 3 — 3 marks

Answer: 0.0265 cm$/s$

$\dfrac{dV}{dh} = \pi h^2$ **A1**

$\dfrac{dh}{dt} = \dfrac{3}{36\pi}$ **M1**

0.0265 cm$/s$ **A1**

Question 4 — 3 marks

Answer: 0.124 cm$/s$

$\dfrac{dV}{dr} = 4\pi r^2$ **A1**

$\dfrac{100}{4\pi(64)}$ **M1**

0.124 cm$/s$ **A1**

Question 5 — 4 marks

Answer:

$f'(x) = (2x - x^2)e^{-x}$; $x = 0,\ 2$

product rule **M1**

$f'(x) = (2x - x^2)e^{-x}$ **A1**

$x(2-x) = 0$ **M1**

$x = 0,\ x = 2$ **A1**

Question 6 — 4 marks

Answer: $\left(e,\ \dfrac{1}{e}\right)$

$\dfrac{1-\ln x}{x^2}$ **A2**

$x = e$ **A1**

$\left(e,\ \dfrac{1}{e}\right)$ **A1**

Question 7 — 4 marks

Answer: $2/27$

$\dfrac{dr}{dt} = \dfrac{1}{3}(2t+1)^{-\frac{2}{3}} \times 2$ *M1*

A1

at $t = 13$:

$\dfrac{2}{3} \times 27^{-\frac{2}{3}} = \dfrac{2}{3} \times \dfrac{1}{9}$ **(M1)**

$\dfrac{2}{27}$ mm per hour **A1**

Question 8 — 4 marks

Answer: $0.3\ \text{m/s}$ downwards

$x^2 + y^2 = 25$ **A1**

$2x\dot{x} + 2y\dot{y} = 0$ **M1**

$y = 4$ **A1**

$\dot{y} = -0.3\ \text{m/s}$, sliding down at $0.3\ \text{m/s}$ **A1**

Question 9 — 8 marks

Answer: $\pi/2, 3\pi/2$

For stationary points, $f'(x) = 0$. **M1**

Use the Chain Rule to differentiate $f(x) = e^{\sin x}$.

Outer derivative $= e^{\sin x}$. Inner derivative $= \cos x$. **M1 A1**

$f'(x) = \cos x \cdot e^{\sin x}$. **A1**

Set to zero: $\cos x \cdot e^{\sin x} = 0$. **M1**

Since $e^{\sin x}$ is an exponential, it is always strictly positive ($e^{\sin x} > 0$). It can never equal zero. **R1 A1**

Therefore, we only need to solve $\cos x = 0$ in the domain $0 \le x \le 2\pi$.

$x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$. **A1**

Question 10 — 4 marks

Answer: (a) 0.424; (b) 4.8

a. $d = \sqrt{2}s$, $\dfrac{dd}{dt} = \sqrt{2}\dfrac{ds}{dt}$ **M1**

$0.424\ \text{cm s}^{-1}$ **A1**

b. $\dfrac{dA}{dt} = 2s\dfrac{ds}{dt} = 2 \times 8 \times 0.3$ **M1**

$4.8\ \text{cm}^2\ \text{s}^{-1}$ **A1**

Every solution is also on ibmathrevision.com/hlai/advanced-differentiation-and-related-rates-exam-practice
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