IB MATHS AA HL · UNIT 3

Vector Lines: exam practice

10 questions · 45 marks · worked solutions and markschemes

Name _____ Date _____ Score / 45

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/vector-lines-exam-practice

1. **2 marks · Paper 1 style · no calculator**

Given that $\mathbf{v} \times \mathbf{w} = \begin{pmatrix} -4 \\ 7 \\ 2 \end{pmatrix}$, write down the exact vector for $\mathbf{w} \times \mathbf{v}$.

2. **3 marks · Paper 1 style · no calculator**

Find the vector product $\mathbf{a} \times \mathbf{b}$ where $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$.

3. **2 marks · Paper 1 style · no calculator**

Find a vector equation of the line through $A(2, -1, 4)$ and $B(5, 1, 1)$.

Find the equation. **[2]**

4. **3 marks · Paper 1 style · no calculator**

Find the acute angle between the lines with directions $\begin{pmatrix}1\\2\\2\end{pmatrix}$ and $\begin{pmatrix}2\\-1\\2\end{pmatrix}$.

Find the angle. **[3]**

5. **5 marks · Paper 1 style · no calculator**

Find the shortest distance from $P(4,1,-2)$ to the line $\mathbf{r}=\begin{pmatrix}1\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\-2\end{pmatrix}$.

Find the distance. **[5]**

6. **5 marks · Paper 1 style · no calculator**

Show that the lines $L_1:\mathbf{r}=\begin{pmatrix}1\\0\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\0\end{pmatrix}$ and $L_2:\mathbf{r}=\begin{pmatrix}0\\3\\0\end{pmatrix}+\mu\begin{pmatrix}0\\1\\1\end{pmatrix}$ are skew.

Show the lines are skew. **[5]**

7. **4 marks · Paper 1 style · no calculator**

The line L has equation $\boldsymbol{r} = \begin{pmatrix}1\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\1\end{pmatrix}$. The point P has coordinates $(0, 4, 3)$.

a. Find the coordinates of the foot of the perpendicular from P to L. **[3]**

b. Hence find the shortest distance from P to L, in exact form. **[1]**

8. **7 marks · Paper 1 style · no calculator**

The points $A(2, 0, 1)$, $B(1, 3, 0)$ and $C(0, 1, 4)$ are given, and O is the origin.

a. Find $\overrightarrow{AB} \times \overrightarrow{AC}$. **[3]**

b. Find the exact area of triangle ABC. **[2]**

c. Find the volume of the tetrahedron $OABC$, given that it equals $\frac{1}{6}\left|\overrightarrow{OA} \cdot (\overrightarrow{OB} \times \overrightarrow{OC})\right|$. **[2]**

9. **7 marks · Paper 2 style · calculator**

The line L_1 has vector equation $\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$, $t \in \mathbb{R}$. The point A has coordinates $(0, 1, 5)$.

a. Find the position vector of the point C, the foot of the perpendicular from A to L_1. [3 marks]

The line L_2 has vector equation $\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, $s \in \mathbb{R}$. The point B lies on L_2.

b. Given that the area of triangle ACB is $\frac{\sqrt{179}}{2}$, find the possible position vectors of B. [4 marks]

10. **7 marks · Paper 1 style · no calculator**

The lines $L_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ and $L_2 : \mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ intersect.

(a) Find the point of intersection. **[4]**

(b) Find the acute angle between the lines. **[3]**

Answers and worked solutions

Question 1

2 marks

Answer: $(4, -7, -2)$

The vector product is anti-commutative:

$\mathbf{w} \times \mathbf{v} = -(\mathbf{v} \times \mathbf{w})$ M1

$\mathbf{w} \times \mathbf{v} = \begin{pmatrix} 4 \\ -7 \\ -2 \end{pmatrix}$ A1

Question 2

3 marks

Answer: $(-1, 7, 5)$

Use the component formula for the vector product:

$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} (-2)(-1) - (3)(1) \\ (3)(2) - (1)(-1) \\ (1)(1) - (-2)(2) \end{pmatrix}$

M1

$= \begin{pmatrix} 2 - 3 \\ 6 + 1 \\ 1 + 4 \end{pmatrix}$ A1

$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} -1 \\ 7 \\ 5 \end{pmatrix}$ A1

Question 3

2 marks

Answer:

$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}$

direction $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}$ A1

$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}$ A1

Question 4

3 marks

Answer: $63.6°$

$\cos\theta = \dfrac{2 - 2 + 4}{3 \times 3}$ $M1A1$

$63.6°$ A1

Question 5

5 marks

Answer: $\dfrac{5\sqrt{2}}{3}$

general point

$Q(1 + \lambda, 2\lambda, 1 - 2\lambda)$ A1

$\overrightarrow{PQ} \cdot \mathbf{d} = 0$ M1

$(\lambda - 3) + 2(2\lambda - 1) - 2(3 - 2\lambda) = 0$

, $\lambda = \dfrac{11}{9}$ A1

$|\overrightarrow{PQ}|$ M1

$\dfrac{5\sqrt{2}}{3}$ A1

Question 6

5 marks

Answer: a proof: the worked solution above is the full answer

not parallel (directions not multiples) R1

$1 + \lambda = 0$, $\lambda = 3 + \mu$, $2 = \mu$ M1

$\lambda = -1$ and $\mu = 2$ A1

second equation: $-1 \neq 5$ A1

no intersection and not parallel, so skew R1 AG

Question 7

4 marks

Answer: (a) $(1/3, 4/3, -1/3)$; (b) $\sqrt{165}/3$

a. $\left(\overrightarrow{OP} - \left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}\right)\right) \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 0$ M1

$\lambda = -\frac{1}{3}$ A1

$\left(\frac{1}{3}, \frac{4}{3}, -\frac{1}{3}\right)$ A1

b. $\frac{\sqrt{165}}{3}$ A1

Question 8

7 marks

Answer: (a) $(10, 5, 5)$; (b) $\sqrt{150}/2$; (c) $25/6$

a. $\overrightarrow{AB} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$,

$\overrightarrow{AC} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$ A1

$\begin{pmatrix} 10 \\ 5 \\ 5 \end{pmatrix}$ $M1A1$

b. $\frac{1}{2}\sqrt{150}$ $M1A1$

c. $\overrightarrow{OB} \times \overrightarrow{OC} = \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$ A1

volume $= \frac{25}{6}$ A1

Question 9

7 marks

Answer: (a) C) $(2/3, -13/6, 19/6)$; (b) B) $(1.82, -2.82, 6.64), (-1.11, 0.114, 0.772)$

(a)

Let C be on L_1:

$\mathbf{c} = \begin{pmatrix} 1 + 2t \\ -2 + t \\ 3 - t \end{pmatrix}$, so

$\vec{AC} = \mathbf{c} - \mathbf{a} = \begin{pmatrix} 1 + 2t \\ -3 + t \\ -2 - t \end{pmatrix}$

$\vec{AC}$ is perpendicular to

$\mathbf{d}_1 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$:

$2(1 + 2t) + (-3 + t) - (-2 - t) = 0$

M1

$1+6t=0 \Rightarrow t=-\frac{1}{6}$ **A1**

$\mathbf{c}=\begin{pmatrix}\frac{2}{3}\\-\frac{13}{6}\\\frac{19}{6}\end{pmatrix}$ **A1**

(b)

$\vec{AC}=\begin{pmatrix}\frac{2}{3}\\-\frac{19}{6}\\-\frac{11}{6}\end{pmatrix}$

B on L_2: $\mathbf{b}=\begin{pmatrix}-1+s\\-s\\1+2s\end{pmatrix}$, so

$\vec{AB}=\mathbf{b}-\mathbf{a}=\begin{pmatrix}s-1\\-s-1\\2s-4\end{pmatrix}$

M1

Area

$=\frac{1}{2}|\vec{AC}\times\vec{AB}|=\frac{\sqrt{179}}{2}$, so

$|\vec{AC}\times\vec{AB}|^2=179$

Compute the vector product:

$\vec{AC}\times\vec{AB}=\frac{1}{6}\begin{pmatrix}-49s+65\\-19s+27\\15s-23\end{pmatrix}$

M1

(e.g. third component:

$\frac{2}{3}(-s-1)+\frac{19}{6}(s-1)=\frac{4(-s-1)+19(s-1)}{6}=\frac{15s-23}{6}$

)

$\frac{1}{36}\left[(-49s+65)^2+(-19s+27)^2+(15s-23)^2\right]=179$

$2987s^2-8086s+5483=6444$

$2987s^2-8086s-961=0$ **A1**

Solve (GDC):

$s=\frac{8086\pm\sqrt{76865424}}{5974}$, so

$s=2.8211$ or $s=-0.11404$

$\mathbf{b}=\begin{pmatrix}1.82\\-2.82\\6.64\end{pmatrix}$ or

$\mathbf{b}=\begin{pmatrix}-1.11\\0.114\\0.772\end{pmatrix}$ (3 s.f.) **A1**

Question 10

7 marks

Answer: (a) $(2,1,2)$; (b) $\frac{\pi}{3}$ ($60°$)

(a)

$1+\lambda=4+2\mu$,

$2-\lambda=2+\mu$ **M1**

$\lambda=1$, $\mu=-1$ **A1**

z: $2(1)=3+(-1)$, consistent **R1**

$(2,1,2)$ **A1**

(b)

$\cos\theta=\frac{2-1+2}{\sqrt{6}\sqrt{6}}$ $M1$**A1**

$\theta=\frac{\pi}{3}$ **A1**

Every solution is also on ibmathrevision.com/hlaa/vector-lines-exam-practice
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