IB MATHS AA HL · UNIT 3

Trigonometric Identities: exam practice

10 questions · 41 marks · worked solutions and markschemes

Name _____ Date _____ Score / 41

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/trigonometric-identities-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Prove that $\dfrac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$.

Prove the identity. **[3]**

2. **3 marks · Paper 1 style · no calculator**

Prove that $(\sin x + \cos x)^2 = 1 + \sin 2x$.

Prove the identity. **[3]**

3. **3 marks · Paper 1 style · no calculator**

Show that $\cos^4 x - \sin^4 x = \cos 2x$.

Show this. **[3]**

4. 3 marks · Paper 1 style · no calculator

Prove that $\dfrac{1-\cos 2x}{\sin 2x} = \tan x$.

Prove the identity. **[3]**

5. 4 marks · Paper 1 style · no calculator

Given $\cos 2\theta = \dfrac{7}{25}$ with $0 < \theta < \dfrac{\pi}{2}$, find the exact values of $\cos\theta$ and $\sin\theta$.

Find both values. **[4]**

6. 4 marks · Paper 1 style · no calculator

Simplify $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x}$ to a single trigonometric ratio.

Simplify. **[4]**

7. 5 marks · Paper 1 style · no calculator

Given that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$, with A and B acute, find the exact value of $\sin(A+B)$ and of $\tan(A-B)$.

Find both values. **[5]**

8. 5 marks · Paper 1 style · no calculator

Show that $\sin 3x = 3\sin x - 4\sin^3 x$, using $\sin 3x = \sin(2x + x)$.

Show this. **[5]**

9. 6 marks · Paper 2 style · calculator

Show that $\cos 2x + \sin x = 0$ can be written as $2\sin^2 x - \sin x - 1 = 0$, and hence solve it for $0 \leq x \leq 2\pi$.

(a) Show the quadratic form. **[2]**

(b) Solve the equation. **[4]**

10. **5 marks · Paper 1 style · no calculator**

Prove that $\tan\left(x+\frac{\pi}{4}\right)=\frac{1+\tan x}{1-\tan x}$ and hence find the exact value of $\tan\frac{7\pi}{12}$.

(a) Prove the identity. **[2]**

(b) Find $\tan\frac{7\pi}{12}$. **[3]**

Answers and worked solutions

Question 1 — 3 marks

Answer: a proof: the worked solution above is the full answer

$$\frac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$$ *M*1A1

$$=\frac{\sin\theta}{\cos\theta}=\tan\theta$$ A1 AG

Question 2 — 3 marks

Answer: a proof: the worked solution above is the full answer

$\sin^2 x+2\sin x\cos x+\cos^2 x$ M1

$=1+2\sin x\cos x$ A1

$=1+\sin 2x$ A1 AG

Question 3 — 3 marks

Answer: a proof: the worked solution above is the full answer

$(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)$ *M*1A1

$=\cos 2x\times 1$ A1 AG

Question 4 — 3 marks

Answer: a proof: the worked solution above is the full answer

$$\frac{2\sin^2 x}{2\sin x\cos x}$$ *M*1A1

$=\tan x$ A1 AG

Question 5 — 4 marks

Answer: $\cos\theta=\frac{4}{5}$, $\sin\theta=\frac{3}{5}$

$2\cos^2\theta-1=\frac{7}{25}$ M1

$\cos\theta=\frac{4}{5}$ A1

$1-2\sin^2\theta=\frac{7}{25}$ M1

$\sin\theta=\frac{3}{5}$ A1

Question 6 — 4 marks

Answer: $2\csc x$

$$\frac{\sin^2 x+(1+\cos x)^2}{\sin x(1+\cos x)}$$ M1

numerator $=2+2\cos x$ A1

$$\frac{2(1+\cos x)}{\sin x(1+\cos x)}$$ A1

$2\csc x$ A1

Question 7 — 5 marks

Answer: $\frac{56}{65}$, $\frac{16}{63}$

$\sin A=\frac{3}{5}$, $\cos A=\frac{4}{5}$,

$\sin B=\frac{5}{13}$, $\cos B=\frac{12}{13}$ A1

$\sin(A+B)=\frac{3}{5}\cdot\frac{12}{13}+\frac{4}{5}\cdot\frac{5}{13}$ M1

$=\frac{56}{65}$ A1

$\tan(A-B)=\frac{\frac{3}{4}-\frac{5}{12}}{1+\frac{15}{48}}$ M1

$=\frac{16}{63}$ A1

Question 8 — 5 marks

Answer: a proof: the worked solution above is the full answer

$\sin 2x\cos x+\cos 2x\sin x$ M1

$2\sin x\cos^2 x+(1-2\sin^2 x)\sin x$ A1A1

$2\sin x(1-\sin^2 x)+\sin x-2\sin^3 x$ M1

$=3\sin x-4\sin^3 x$ A1 AG

Question 9 — 6 marks

Answer: (b)

$x=\frac{\pi}{2},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$

(a)

$1-2\sin^2 x+\sin x=0$ M1

$\times(-1)$:

$2\sin^2 x-\sin x-1=0$ A1

AG

(b)

$(2\sin x+1)(\sin x-1)=0$

M1

$x=\frac{\pi}{2}$ A1

$x=\frac{7\pi}{6},\ \frac{11\pi}{6}$ A1A1

Question 10 — 5 marks

Answer: (b) $-2-\sqrt{3}$

(a)

$$\frac{\tan x+\tan\frac{\pi}{4}}{1-\tan x\tan\frac{\pi}{4}}$$ M1

$\tan\frac{\pi}{4}=1$ A1 AG

(b)

$x=\frac{\pi}{3}$: $\frac{1+\sqrt{3}}{1-\sqrt{3}}$ *M*1A1

$=-2-\sqrt{3}$ A1

Every solution is also on ibmathrevision.com/hlaa/trigonometric-identities-exam-practice
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