Trigonometric Functions: exam practice

10 questions · 34 marks · worked solutions and markschemes

Name _____ Date _____ Score / 34

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/trigonometric-functions-exam-practice

1. **2 marks · Paper 1 style · no calculator**

Write down the period of each function: $f(x) = \tan\frac{x}{3}$, $g(x) = 2\cos(4x)$.

Write down both periods. **[2]**

2. **3 marks · Paper 1 style · no calculator**

Let $f(x) = 3\sin\left(2x - \frac{\pi}{2}\right) + 1$.

Write down the amplitude, the period and the range of f. **[3]**

3. **3 marks · Paper 1 style · no calculator**

The graph of $y = a\sin(bx) + c$ has a maximum at $\left(\frac{\pi}{6},\ 7\right)$ and the next minimum at $\left(\frac{\pi}{2},\ -1\right)$, with $a, b > 0$.

Find a, b and c. **[3]**

4. **3 marks · Paper 1 style · no calculator**

The graph of $y = \cos x$ is transformed to $y = 2\cos\left(x - \frac{\pi}{3}\right) - 1$.

Find the coordinates of the maximum point with $0 \leq x \leq 2\pi$ and the minimum value. **[3]**

5. **3 marks · Paper 2 style · calculator**

Let $f(x) = 4\sin(3x)$ for $0 \leq x \leq \pi$.

Find the number of solutions of $f(x) = 2$ and the smallest one. **[3]**

6. **3 marks · Paper 1 style · no calculator**

The function $f(x) = p\cos(qx)$ has range $-5 \leq f(x) \leq 5$ and period $\frac{2\pi}{5}$, with $p, q > 0$.

Write down p and q, and find $f\left(\frac{\pi}{15}\right)$. **[3]**

7. **3 marks · Paper 1 style · no calculator**

Let $g(x) = \tan\left(x + \frac{\pi}{4}\right)$ for $0 \leq x \leq \pi$.

Find the equation of the vertical asymptote and the x-intercept. **[3]**

8. 4 marks · Paper 2 style · calculator

The depth of water in a harbour is $d(t) = a\cos(bt) + c$ metres, t hours after midnight. High tide is 9.4 m at midnight and the next low tide is 2.6 m at $06{:}15$.

Find a, b and c. **[4]**

9. 5 marks · Paper 2 style · calculator

A Ferris wheel of diameter 40 m turns once every 8 minutes. A seat's height above the ground is $h(t) = 22 - 20\cos\left(\dfrac{\pi t}{4}\right)$, t in minutes.

(a) Find the height of the seat after 3 minutes. **[2]**

(b) Find how long, in each turn, the seat is more than 30 m above the ground. **[3]**

10. 5 marks · Paper 2 style · calculator

The number of hours of daylight in a city is modelled by $D(t) = 12 + 3.5\sin\left(\dfrac{2\pi}{365}(t - 80)\right)$, where t is the day of the year.

(a) Find the maximum number of hours of daylight and the day it occurs. **[2]**

(b) Find the number of days with more than 14 hours of daylight. **[3]**

Answers and worked solutions

Question 1 — 2 marks

Answer: 3π, $\frac{\pi}{2}$

3π **A1**

$\frac{\pi}{2}$ **A1**

Question 2 — 3 marks

Answer: amplitude 3, period π, $-2 \le f(x) \le 4$

amplitude 3 **A1**

period π **A1**

$-2 \le f(x) \le 4$ **A1**

Question 3 — 3 marks

Answer: $a = 4$, $b = 3$, $c = 3$

$a = 4$ **A1**

$c = 3$ **A1**

period $\frac{2\pi}{3}$, so $b = 3$ **A1**

Question 4 — 3 marks

Answer: $\left(\frac{\pi}{3},\ 1\right)$, minimum -3

maximum when $x - \frac{\pi}{3} = 0$ **M1**

$\left(\frac{\pi}{3},\ 1\right)$ **A1**

minimum -3 **A1**

Question 5 — 3 marks

Answer: 4 solutions; $x = \frac{\pi}{18}$

$\sin 3x = \frac{1}{2}$ **M1**

4 solutions **A1**

smallest $x = \frac{\pi}{18}$ **A1**

Question 6 — 3 marks

Answer: $p = 5$, $q = 5$, $f\left(\frac{\pi}{15}\right) = \frac{5}{2}$

$p = 5$ **A1**

$q = 5$ **A1**

$5\cos\frac{\pi}{3} = \frac{5}{2}$ **A1**

Question 7 — 3 marks

Answer: $x = \frac{\pi}{4}$; $\left(\frac{3\pi}{4},\ 0\right)$

$x + \frac{\pi}{4} = \frac{\pi}{2}$ **M1**

$x = \frac{\pi}{4}$ **A1**

$x-$intercept $\frac{3\pi}{4}$ **A1**

Question 8 — 4 marks

Answer: $a = 3.4$, $b = 0.503$, $c = 6$

$a = 3.4$ **A1**

$c = 6$ **A1**

period 12.5 hours **M1**

$b = \frac{2\pi}{12.5} = 0.503$ **A1**

Question 9 — 5 marks

Answer: (a) $36.1\,m$; (b) 2.95 minutes

(a)

$22 - 20\cos\frac{3\pi}{4}$ **M1**

$36.1\,m$ **A1**

(b)

$\cos\frac{\pi t}{4} < -0.4$ **M1**

t from 2.52 to 5.48 **A1**

2.95 min **A1**

Question 10 — 5 marks

Answer: (a) 15.5 hours on day 171; (b) 112 days

(a)

15.5 hours **A1**

$t = 80 + 91.25 = 171$ **A1**

(b)

$\sin(\ldots) > \frac{4}{7}$ **M1**

t from 115.3 to 227.2 **A1**

112 days **A1**

Every solution is also on ibmathrevision.com/hlaa/trigonometric-functions-exam-practice
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