IB MATHS AA HL · UNIT 2

Transformations and Modulus: exam practice

10 questions · 38 marks · worked solutions and markschemes

Name _____ Date _____ Score / 38

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/transformations-and-modulus-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Solve the absolute value equation $|4x - 1| = 11$ algebraically.

2. **3 marks · Paper 1 style · no calculator**

A function $f(x)$ has x-intercepts at $x = -2$ and $x = 7$. State the equations of the vertical asymptotes of the reciprocal graph $y = \frac{1}{f(x)}$.

3. **2 marks · Paper 1 style · no calculator**

The graph of $y = f(x)$ is transformed to $y = 2f(x - 1) + 3$. The point $(4, -2)$ is on $y = f(x)$. Find the image of this point. **[2]**

4. **3 marks · Paper 1 style · no calculator**

Solve $|3x - 2| \geq 4$.

Solve the inequality. **[3]**

5. **3 marks · Paper 2 style · calculator**

Solve $|x^2 - 5| = 4$.

Find all x. **[3]**

6. **3 marks · Paper 1 style · no calculator**

The graph of $y = x^2$ is mapped to $y = -\frac{1}{2}(x + 4)^2$.

Describe the transformations, in order. **[3]**

7. **5 marks · Paper 2 style · calculator**

Solve the quadratic modulus equation $|x^2 - 3x| = 4$.

8. **4 marks · Paper 1 style · no calculator**

The graph of $y = \ln x$ is reflected in the x-axis, then stretched horizontally by a factor of 2, then translated 3 units up.

Find the equation of the final graph, and its x-intercept. **[4]**

9. **7 marks · Paper 2 style · calculator**

Find the exact area of the triangle formed by the intersection of the modulus graph $y = 8 - |2x - 6|$ and the x-axis.

10. **5 marks · Paper 1 style · no calculator**

The graph of $y = f(x)$ has a horizontal asymptote of $y = 3$ and a vertical asymptote of $x = -4$. Find the equations of the horizontal and vertical asymptotes of the transformed function $g(x) = -2f(x - 5) + 1$.

Answers and worked solutions

Question 1 — 3 marks

Answer: $3, -5/2$

1. Two linear cases:
   $4x - 1 = 11$ or
   $4x - 1 = -11$ **M1**
   $4x = 12 \Rightarrow x = 3$ **A1**
   $4x = -10 \Rightarrow x = -\frac{5}{2}$ **A1**

Question 2 — 3 marks

Answer: $x = -2, x = 7$

1. Recognizing that vertical asymptotes of $y = \frac{1}{f(x)}$ occur at values where $f(x) = 0$ **(M1)**
   Identifying that the roots of $f(x)$ are given by its $x-$intercepts **(R1)**
   The equations of the vertical asymptotes are $x = -2, x = 7$ **A1**

Question 3 — 2 marks

Answer: $(5, -1)$

x: $4 + 1 = 5$ **A1**

y: $2(-2) + 3 = -1$ **A1**

Question 4 — 3 marks

Answer: $x \leq -\frac{2}{3}$ or $x \geq 2$

$3x - 2 \geq 4$ or $3x - 2 \leq -4$ **M1**

$x \geq 2$ **A1**

$x \leq -\frac{2}{3}$ **A1**

Question 5 — 3 marks

Answer: $x = \pm 1, x = \pm 3$

$x^2 - 5 = 4$ or $x^2 - 5 = -4$ **M1**

$x = \pm 3$ **A1**

$x = \pm 1$ **A1**

Question 6 — 3 marks

Answer: stretch factor $\frac{1}{2}$, reflect in the $x-$axis, translate by $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$

vertical stretch, scale factor $\frac{1}{2}$ **A1**

reflection in the $x-$axis **A1**

translation $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ **A1**

Question 7 — 5 marks

Answer: $4, -1$

1. Set up two equations:
   $x^2 - 3x = 4$ or
   $x^2 - 3x = -4$ **M1**
   Solve $x^2 - 3x - 4 = 0$:
   $(x - 4)(x + 1) = 0$ **(M1)**
   $x = 4$ or $x = -1$ **A1**
   $x^2 - 3x + 4 = 0$ has discriminant $\Delta = 9 - 16 = -7 < 0$, so no real roots **R1**
   Solutions: $x = 4,\ x = -1$ **A1**

Question 8 — 4 marks

Answer: $y = 3 - \ln \frac{x}{2}$;

$x = 2e^3$

$y = -\ln x$ **A1**

$y = -\ln \frac{x}{2}$ **A1**

$y = 3 - \ln \frac{x}{2}$ **A1**

$x-$intercept $2e^3$ **A1**

Question 9 — 7 marks

Answer: 32

1. Set $8 - |2x - 6| = 0$, so
   $2x - 6 = \pm 8$ **M1**
   $x = 7$ **A1**
   $x = -1$ **A1**
   Vertex where $2x - 6 = 0$ **M1**
   Vertex $(3, 8)$, so height $= 8$ **A1**
   Base $= 7 - (-1) = 8$;
   area $= \frac{1}{2} \times 8 \times 8$ **M1**
   Area $= 32$ **A1**

Question 10 — 5 marks

Answer: (Vertical asymptote: $x =$) 1; (Horizontal asymptote: $y =$) -5

1. Vertical asymptote is translated 5 units right:
   $x = -4 + 5$ **(M1)**
   $x = 1$ **A1**
   Horizontal asymptote: vertical stretch by factor -2 **(M1)**
   $y = -2(3) = -6$ **(A1)**
   then translate up 1:
   $y = -6 + 1 = -5$ **A1**

Every solution is also on ibmathrevision.com/hlaa/transformations-and-modulus-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.