IB MATHS AA HL · UNIT 3

The unit circle and exact values of trigonometric ratios: exam practice

10 questions · 29 marks · worked solutions and markschemes

Name _____ Date _____ Score / 29

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/the-unit-circle-and-exact-values-of-trigonometric-ratios-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Write down the exact values of $\sin\frac{4\pi}{3}$, $\cos\frac{7\pi}{4}$ and $\tan\frac{5\pi}{6}$.

Write down the values. **[3]**

2. **2 marks · Paper 1 style · no calculator**

Find all values of θ in $[0, 2\pi]$ for which $\cos\theta = -\frac{\sqrt{3}}{2}$.

Find θ. **[2]**

3. **2 marks · Paper 1 style · no calculator**

Find the exact value of $\cos\left(-\frac{2\pi}{3}\right)+\sin\frac{11\pi}{6}$.

Find the value. **[2]**

4. **3 marks · Paper 1 style · no calculator**

Given that $\cos\theta=-\frac{3}{5}$ and $\pi<\theta<\frac{3\pi}{2}$, find the exact values of $\sin\theta$ and $\tan\theta$.

Find both values. **[3]**

5. **3 marks · Paper 1 style · no calculator**

Find the exact value of $\sin^2\frac{\pi}{3}+\cos\frac{5\pi}{4}\sin\frac{3\pi}{4}$.

Find the value. **[3]**

6. **3 marks · Paper 1 style · no calculator**

The point $P\left(-\frac{5}{13},\ k\right)$ lies on the unit circle, in the third quadrant. OP makes an angle θ with the positive x-axis.

Find k and write down $\tan\theta$. **[3]**

7. 3 marks · Paper 1 style · no calculator

Given that $\tan\alpha = -\frac{3}{4}$ and $\frac{3\pi}{2} < \alpha < 2\pi$, find the exact value of $\sin\alpha$.

Find $\sin\alpha$. **[3]**

8. 3 marks · Paper 1 style · no calculator

Given that $\sin\theta = k$ with $0 < \theta < \frac{\pi}{2}$, express in terms of k: $\sin(\pi - \theta)$, $\cos\left(\frac{\pi}{2} + \theta\right)$ and $\sin(\theta + \pi)$.

Write each one in terms of k. **[3]**

9. 4 marks · Paper 1 style · no calculator

Find the exact value of $\tan\frac{\pi}{8}$, using $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$ with $A = \frac{\pi}{8}$.

Find the value. **[4]**

10. 3 marks · Paper 1 style · no calculator

Use $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ to find the exact value of $\cos\frac{\pi}{12}$.

Find the value. **[3]**

Answers and worked solutions

Question 1 — 3 marks

Answer: $-\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}, -\frac{\sqrt{3}}{3}$

$-\frac{\sqrt{3}}{2}$ A1

$\frac{\sqrt{2}}{2}$ A1

$-\frac{\sqrt{3}}{3}$ A1

Question 2 — 2 marks

Answer: $\frac{5\pi}{6}, \frac{7\pi}{6}$

$\frac{5\pi}{6}$ A1

$\frac{7\pi}{6}$ A1

Question 3 — 2 marks

Answer: -1

$-\frac{1}{2} - \frac{1}{2}$ A1

-1 A1

Question 4 — 3 marks

Answer: $\sin\theta = -\frac{4}{5}$, $\tan\theta = \frac{4}{3}$

$\sin^2\theta = \frac{16}{25}$ M1

$\sin\theta = -\frac{4}{5}$ A1

$\tan\theta = \frac{4}{3}$ A1

Question 5 — 3 marks

Answer: $\frac{1}{4}$

$\frac{3}{4}$ A1

$-\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = -\frac{1}{2}$ A1

$\frac{1}{4}$ A1

Question 6 — 3 marks

Answer: $k = -\frac{12}{13}$, $\tan\theta = \frac{12}{5}$

$k^2 = 1 - \frac{25}{169}$ M1

$k = -\frac{12}{13}$ A1

$\tan\theta = \frac{12}{5}$ A1

Question 7 — 3 marks

Answer: $-\frac{3}{5}$

$\sec^2\alpha = 1 + \frac{9}{16}$, $\cos\alpha = \frac{4}{5}$ M1

$\sin\alpha < 0$ in the fourth quadrant R1

$-\frac{3}{5}$ A1

Question 8 — 3 marks

Answer: k, $-k$, $-k$

k A1

$-k$ A1

$-k$ A1

Question 9 — 4 marks

Answer: $\sqrt{2} - 1$

$1 = \frac{2t}{1 - t^2}$ M1

$t^2 + 2t - 1 = 0$ A1

$t = -1 \pm \sqrt{2}$ A1

$t > 0$, so $\sqrt{2} - 1$ A1

Question 10 — 3 marks

Answer: $\frac{\sqrt{2} + \sqrt{6}}{4}$

$\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$ M1

$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$ A1

$\frac{\sqrt{2} + \sqrt{6}}{4}$ A1

Every solution is also on ibmathrevision.com/hlaa/the-unit-circle-and-exact-values-of-trigonometric-ratios-exam-practice
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