IB MATHS AA HL · UNIT 1

Systems of Equations: exam practice

10 questions · 48 marks · worked solutions and markschemes

Name _____ Date _____ Score / 48

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/systems-of-equations-exam-practice

1. **2 marks · Paper 2 style · calculator**

Use technology to solve $1.2x + y - z = 4.3$, $x - 0.5y + 2z = 0.9$, $3x + 2y + z = 7.7$.

2. **4 marks · Paper 2 style · calculator**

The quadratic $y = ax^2 + bx + c$ passes through $(1, 4)$, $(2, 11)$ and $(-1, 2)$.

Find a, b and c. **[4]**

3. **4 marks · Paper 2 style · calculator**

A shop sells pens, pencils and rulers. 2 pens, 3 pencils and 1 ruler cost $\$7.10$; 1 pen, 4 pencils and 2 rulers cost $\$7.20$; 3 pens, 1 pencil and 3 rulers cost $\$9.50$.

Find the cost of each item. **[4]**

4. **5 marks · Paper 1 style · no calculator**

Solve the system $x - y + 2z = 5$, $3x + y - z = 2$, $2x + 2y + z = 9$ by elimination.

Find x, y and z. **[5]**

5. **7 marks · Paper 2 style · calculator**

Three planes are given by the equations:

$$2x + y + 3z = 1$$
$$-x + 2y + z = p$$
$$3x - y + 2z = 4$$

Find the exact value of the parameter p such that the three planes intersect in a single common line.

6. **3 marks · Paper 2 style · calculator**

Solve the following system of linear equations using your GDC:

$$2x - 3y + z = 10$$
$$x + 4y - 2z = -3$$
$$-3x + y + 5z = 7$$

7. **6 marks · Paper 1 style · no calculator**

Solve the following system of equations algebraically:

$$\begin{aligned} x + y + z &= 6 \\ 2x - y + 3z &= 9 \\ 3x + 2y - z &= 4 \end{aligned}$$

8. **5 marks · Paper 1 style · no calculator**

Consider the system $x + 2y - z = 3$, $2x - y + z = 1$, $3x + y + kz = 4$.

(a) Find the value of k for which the system does not have a unique solution. **[3]**

(b) For this value of k, show that the system has infinitely many solutions. **[2]**

9. **6 marks · Paper 1 style · no calculator**

Consider the system of linear equations in x, y, z with real parameters a and b:

$$Eq_1 : x - 2y + z = 2$$
$$Eq_2 : 2x + y - 3z = 5$$
$$Eq_3 : 4x - 3y - z = a + b$$
$$Eq_4 : x + 3y - 4z = a - b$$

Find the exact values of a and b required for this overdetermined system to be consistent, given the hint that Equation 3 and Equation 4 are linear combinations of Equation 1 and Equation 2.

10. **6 marks · Paper 1 style · no calculator**

Consider the system $x + y + z = 2$, $x - y + 2z = 1$, $2x + 3z = k$.

(a) Show that the system has no solution unless $k = 3$. **[3]**

(b) For $k = 3$, find the general solution, using $z = \lambda$. **[3]**

Answers and worked solutions

Question 1

2 marks

Answer:

$x = 2.69, y = 0.231, z = -0.838$

$x = 2.69,\ y = 0.231,\ z = -0.838$ A2

Question 2

4 marks

Answer: $a = 2$, $b = 1$, $c = 1$

$a + b + c = 4$, $4a + 2b + c = 11$, $a - b + c = 2$ $M1$A1

$a = 2$, $b = 1$, $c = 1$ A2

Question 3

4 marks

Answer: pen $\$1.80$, pencil $\$0.80$, ruler $\$1.10$

$2p + 3q + r = 7.1$,
$p + 4q + 2r = 7.2$,
$3p + q + 3r = 9.5$ $M1$A1

pen $\$1.80$, pencil $\$0.80$, ruler $\$1.10$ A2

Question 4

5 marks

Answer: $x = 1$, $y = 2$, $z = 3$

eliminate y: $4x + z = 7$ $M1$A1

$4x + 5z = 19$ A1

$z = 3$, $x = 1$ A1

$y = 2$ A1

Question 5

7 marks

Answer: -3

Find the determinant of the coefficient matrix:

$2(4 + 1) - 1(-2 - 3) + 3(1 - 6)$ M1

$= 10 + 5 - 15 = 0$, so the planes never meet in a single point, for any p A1

Eliminate x using row 1, e.g.

$R_2 + \frac{1}{2}R_1$ and $R_3 - \frac{3}{2}R_1$ M1

$R_2 + \frac{1}{2}R_1$: $2.5y + 2.5z = p + 0.5$ A1

$R_3 - \frac{3}{2}R_1$: $-2.5y - 2.5z = 2.5$, i.e. $2.5y + 2.5z = -2.5$ A1

For a common line the two equations must be the same (consistent):

$p + 0.5 = -2.5$ M1

$p = -3$ A1

Question 6

3 marks

Answer: $(x)\ 31/9$; $(y)\ 1/9$; $(z)\ 31/9$

1. Write the system as a matrix equation $AX = B$ with

$$A = \begin{pmatrix} 2 & -3 & 1 \\ 1 & 4 & -2 \\ -3 & 1 & 5 \end{pmatrix},$$

$$B = \begin{pmatrix} 10 \\ -3 \\ 7 \end{pmatrix}$$ (M1)

Solve with the GDC (e.g. $X = A^{-1}B$ or the simultaneous equation solver) (A1)

$x = 3.44,\ y = 0.111,\ z = 3.44$ (exactly $x = z = \frac{31}{9}$, $y = \frac{1}{9}$) A1

Question 7

6 marks

Answer: $(x)\ 1$; $(y)\ 2$; $(z)\ 3$

1. Add the first two equations to eliminate y: $3x + 4z = 15$ M1 A1

Add twice the second equation to the third: $7x + 5z = 22$ A1

Solve these: $21x + 28z = 105$ and $21x + 15z = 66$, so $13z = 39$, $z = 3$ A1

Substitute back: $3x + 12 = 15$, so $x = 1$; then $1 + y + 3 = 6$, so $y = 2$ (M1)

$x = 1,\ y = 2,\ z = 3$ A1

Question 8

5 marks

Answer: (a) 0

(a)

no unique solution when the determinant of the coefficients is 0 M1

$\det = -5k = 0$ A1

$k = 0$ A1

(b)

$R_1 + R_2$: $3x + y = 4$, the same as R_3 M1

consistent, so infinitely many solutions A1 AG

Question 9

6 marks

Answer: (a) 6; (b) 3

Form $2(Eq_1) + Eq_2$:

$2(x - 2y + z) + (2x + y - 3z) = 4x - 3y - z$, the LHS of Eq_3 M1

Match right-hand sides: $2(2) + 5 = 9$, so $a + b = 9$ A1

Form $Eq_2 - Eq_1$:

$(2x + y - 3z) - (x - 2y + z) = x + 3y - 4z$, the LHS of Eq_4 M1

Match right-hand sides: $5 - 2 = 3$, so $a - b = 3$ A1

Solve simultaneously: adding gives $2a = 12$ M1

$a = 6,\ b = 3$ A1

Question 10

6 marks

Answer: (b) $x = \dfrac{3 - 3\lambda}{2}$, $y = \dfrac{1 + \lambda}{2}$, $z = \lambda$

(a)

$R_1 + R_2$: $2x + 3z = 3$ $M1$A1

so $k \neq 3$ gives a contradiction R1 AG

(b)

$x = \dfrac{3 - 3\lambda}{2}$ A1

$y = \dfrac{1 + \lambda}{2}$ A1

$z = \lambda$ A1

Every solution is also on ibmathrevision.com/hlaa/systems-of-equations-exam-practice
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