Reciprocal and Inverse Trig Functions: exam practice

10 questions · 44 marks · worked solutions and markschemes

Name _____ Date _____ Score / 44

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/reciprocal-and-inverse-trig-functions-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Find the exact value of $\sec\left(\frac{\pi}{3}\right) + \csc\left(\frac{\pi}{6}\right)$.

2. **4 marks · Paper 1 style · no calculator**

Find the exact value of the derivative of the function $f(x) = \arcsin(3x)$ at the point where $x = \frac{1}{6}$.

3. **4 marks · Paper 1 style · no calculator**

Find the exact value of $\sin\left(\arccos\frac{2}{3}\right)$ and of $\tan\left(\arcsin\left(-\frac{3}{5}\right)\right)$.

Find both values. **[4]**

4. 4 marks · Paper 2 style · calculator

Let $f(x) = 5\cos x - 12\sin x$.

Find the maximum and minimum values of $\dfrac{1}{20 + f(x)}$. **[4]**

5. 5 marks · Paper 1 style · no calculator

Given that $\sin A = \frac{3}{5}$ where A is an acute angle, and $\cos B = -\frac{5}{13}$ where B is an obtuse angle, find the exact value of $\sin(A + B)$.

6. 5 marks · Paper 1 style · no calculator

Solve the equation $\arctan(x) + \arctan(2x) = \frac{\pi}{4}$ for the exact positive real value of x.

7. **5 marks · Paper 1 style · no calculator**

Solve $\sec^2 x - 3\tan x + 1 = 0$ for $0 \leq x \leq \pi$.

Find x. **[5]**

8. **4 marks · Paper 1 style · no calculator**

Show that $\arctan \frac{1}{2} + \arctan \frac{1}{3} = \frac{\pi}{4}$.

Show this. **[4]**

9. **5 marks · Paper 2 style · calculator**

Solve $\sqrt{3}\cos x - \sin x = 1$ for $0 \leq x \leq 2\pi$, by writing the left side in the form $R\cos(x + \alpha)$.

Find x. **[5]**

10. **5 marks · Paper 2 style · calculator**

Write $3\sin x + 4\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(a) Find R and α. **[3]**

(b) Hence write down the maximum value of $3\sin x + 4\cos x$ and the smallest positive x at which it occurs. **[2]**

Answers and worked solutions

Question 1

3 marks

Answer: 4

Use $\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$

: $\frac{1}{\cos(\pi/3)} + \frac{1}{\sin(\pi/6)}$ M1

$= \frac{1}{1/2} + \frac{1}{1/2} = 2 + 2$ A1

$= 4$ A1

Question 2

4 marks

Answer: $2\sqrt{3}$

Apply the chain rule to the derivative of $\arcsin$:

$f'(x) = \frac{1}{\sqrt{1-(3x)^2}} \times 3$ M1

$f'(x) = \frac{3}{\sqrt{1-9x^2}}$ A1

Substitute $x = \frac{1}{6}$:

$f'\left(\frac{1}{6}\right) = \frac{3}{\sqrt{1-\frac{9}{36}}} = \frac{3}{\sqrt{\frac{3}{4}}}$ M1

$= \frac{3}{\frac{\sqrt{3}}{2}} = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ A1

Question 3

4 marks

Answer: $\frac{\sqrt{5}}{3}, -\frac{3}{4}$

right triangle: $\sqrt{9-4} = \sqrt{5}$ M1

$\frac{\sqrt{5}}{3}$ A1

angle in $\left(-\frac{\pi}{2}, 0\right)$, $\cos = \frac{4}{5}$ M1

$-\frac{3}{4}$ A1

Question 4

4 marks

Answer: maximum $\frac{1}{7}$, minimum $\frac{1}{33}$

$f(x) = 13\cos(x+\alpha)$ A1

$-13 \le f(x) \le 13$ A1

maximum $\frac{1}{7}$ A1

minimum $\frac{1}{33}$ A1

Question 5

5 marks

Answer: $33/65$

A acute: $\cos A = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$ A1

B obtuse, so $\sin B > 0$:

$\sin B = \sqrt{1-\frac{25}{169}} = \frac{12}{13}$ A1

Use $\sin(A+B) = \sin A\cos B + \cos A\sin B = \frac{3}{5}\left(-\frac{5}{13}\right) + \frac{4}{5}\cdot\frac{12}{13}$ M1

$= -\frac{15}{65} + \frac{48}{65}$ A1

$= \frac{33}{65}$ A1

Question 6

5 marks

Answer: $(\sqrt{17}-3)/4$

Take tan of both sides with the compound angle identity:

$\frac{x+2x}{1-2x^2} = \tan\frac{\pi}{4} = 1$ M1

$3x = 1-2x^2 \implies 2x^2+3x-1=0$ A1

$x = \frac{-3\pm\sqrt{9+8}}{4} = \frac{-3\pm\sqrt{17}}{4}$ A1

The negative root makes both arctangents negative, so the sum cannot be $\frac{\pi}{4}$; reject it R1

$x = \frac{\sqrt{17}-3}{4}$ A1

Question 7

5 marks

Answer: $x = \frac{\pi}{4}, x = \arctan 2$

$1 + \tan^2 x - 3\tan x + 1 = 0$ M1

$(\tan x - 1)(\tan x - 2) = 0$ A1

$x = \frac{\pi}{4}$ A1

$x = \arctan 2$ A1A1

Question 8

4 marks

Answer: a proof: the worked solution above is the full answer

$\tan(A+B) = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}}$ M1A1

$= 1$ A1

$0 < A+B < \frac{\pi}{2}$, so

$A+B = \frac{\pi}{4}$ R1 AG

Question 9

5 marks

Answer: $x = \frac{\pi}{6}, x = \frac{3\pi}{2}$

$R = 2, \alpha = \frac{\pi}{6}$ A1A1

$\cos\left(x+\frac{\pi}{6}\right) = \frac{1}{2}$ M1

$x = \frac{\pi}{6}$ A1

$x = \frac{3\pi}{2}$ A1

Question 10

5 marks

Answer: (a) $5\sin(x+0.927)$; (b) 5 at $x = 0.644$

(a)

$R = 5$ A1

$\tan\alpha = \frac{4}{3}$ M1

$\alpha = 0.927$ A1

(b)

5 A1

$x = \frac{\pi}{2} - 0.927 = 0.644$ A1

Every solution is also on ibmathrevision.com/hlaa/reciprocal-and-inverse-trig-functions-exam-practice
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