IB MATHS AA HL · UNIT 2

Rational and Reciprocal Functions: exam practice

10 questions · 37 marks · worked solutions and markschemes

Name _____ Date _____ Score / 37

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/rational-and-reciprocal-functions-exam-practice

1. **2 marks · Paper 1 style · no calculator**

Let $f(x) = \dfrac{3x+4}{2x-1}$.

Write down the equations of the asymptotes of the graph of f. **[2]**

2. **3 marks · Paper 1 style · no calculator**

Solve $|x-4| = 2x+1$.

Find x. **[3]**

3. **3 marks · Paper 2 style · calculator**

Let $g(x) = \dfrac{2x^2 - x + 1}{x-1}$.

Find the equation of the oblique asymptote of the graph of g. **[3]**

4. **4 marks · Paper 1 style · no calculator**

The graph of $y = \dfrac{ax + b}{x + c}$ has asymptotes $x = -2$ and $y = 3$ and passes through $(1, 2)$.

Find a, b and c. **[4]**

5. **3 marks · Paper 1 style · no calculator**

Let $f(x) = \dfrac{x - 3}{x^2 - x - 6}$, $x \neq 3$, $x \neq -2$.

(a) Simplify $f(x)$. **[2]**

(b) Write down the equation of the vertical asymptote of the graph of f. **[1]**

6. **4 marks · Paper 2 style · calculator**

Let $f(x) = \dfrac{6}{x - 2} - 3$.

(a) Find the intercepts of the graph with the axes. **[2]**

(b) Solve $f(x) > 0$. **[2]**

7. **4 marks · Paper 1 style · no calculator**

Let $f(x) = 2x - 6$.

(a) For the graph of $y = |f(x)|$, write down the coordinates of the point where it meets the x-axis and the y-axis. **[2]**

(b) For the graph of $y = f(|x|)$, write down the x-intercepts. **[2]**

8. **4 marks · Paper 2 style · calculator**

Let $f(x) = \dfrac{2x}{x^2 + 1}$.

(a) Write down the equation of the horizontal asymptote. **[1]**

(b) Find the range of f. **[3]**

9. **5 marks · Paper 1 style · no calculator**

Let $f(x) = x^2 - 4x + 3$. Consider the graph of $y = \dfrac{1}{f(x)}$.

(a) Find the vertical asymptotes. **[2]**

(b) Find the turning point of $y = \dfrac{1}{f(x)}$ and state its nature. **[3]**

10. **5 marks · Paper 1 style · no calculator**

Let $f(x) = \dfrac{x^2 + 3x - 2}{x + 1}$.

(a) Write $f(x)$ in the form $x + a + \dfrac{b}{x+1}$. **[3]**

(b) Write down the equations of the asymptotes. **[2]**

Answers and worked solutions

Question 1 — 2 marks

Answer: $x = \frac{1}{2}$, $y = \frac{3}{2}$

$x = \frac{1}{2}$ **A1**

$y = \frac{3}{2}$ **A1**

Question 2 — 3 marks

Answer: 1

$x - 4 = 2x + 1$ gives $x = -5$, rejected ($2x + 1 < 0$) $M1$**R1**

$-(x - 4) = 2x + 1$ gives $x = 1$ **A1**

Question 3 — 3 marks

Answer: $y = 2x + 1$

long division **M1**

$g(x) = 2x + 1 + \frac{2}{x - 1}$ **A1**

$y = 2x + 1$ **A1**

Question 4 — 4 marks

Answer: $a = 3$, $b = 3$, $c = 2$

$c = 2$ **A1**

$a = 3$ **A1**

$2 = \frac{3 + b}{3}$ **M1**

$b = 3$ **A1**

Question 5 — 3 marks

Answer: (a) $\frac{1}{x + 2}$; (b) $x = -2$

(a)

$\frac{x - 3}{(x - 3)(x + 2)}$ **M1**

$\frac{1}{x + 2}$ **A1**

(b)

$x = -2$ **A1**

Question 6 — 4 marks

Answer: (a) $(0, -6)$ and $(4, 0)$; (b) $2 < x < 4$

(a)

$(0, -6)$ **A1**

$(4, 0)$ **A1**

(b)

between the asymptote and the root **M1**

$2 < x < 4$ **A1**

Question 7 — 4 marks

Answer: (a) $(3, 0)$ and $(0, 6)$; (b) $x = \pm 3$

(a)

$(3, 0)$ **A1**

$(0, 6)$ **A1**

(b)

$f(|x|) = 2|x| - 6 = 0$ **M1**

$x = \pm 3$ **A1**

Question 8 — 4 marks

Answer: (a) $y = 0$; (b) $-1 \leq f(x) \leq 1$

(a)

$y = 0$ **A1**

(b)

turning points at $x = \pm 1$ **M1**

$f(1) = 1$, $f(-1) = -1$ **A1**

$-1 \leq f(x) \leq 1$ **A1**

Question 9 — 5 marks

Answer: (a) $x = 1$, $x = 3$; (b) $(2, -1)$, a local maximum

(a)

$f(x) = 0$ **M1**

$x = 1$, $x = 3$ **A1**

(b)

f has minimum $(2, -1)$ **A1**

so $\frac{1}{f}$ has $(2, -1)$ **A1**

a local maximum **A1**

Question 10 — 5 marks

Answer: (a) $x + 2 - \frac{4}{x + 1}$; (b) $x = -1$, $y = x + 2$

(a)

long division **M1**

$x + 2 - \frac{4}{x + 1}$ **A1A1**

(b)

$x = -1$ **A1**

$y = x + 2$ **A1**

Every solution is also on ibmathrevision.com/hlaa/rational-and-reciprocal-functions-exam-practice
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