Probability & Statistics: exam practice

10 questions · 35 marks · worked solutions and markschemes

Name _____ Date _____ Score / 35

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/probability-and-statistics-exam-practice

1. **2 marks · Paper 2 style · calculator**

A random variable X has mean $E(X) = 5$ and variance $\mathrm{Var}(X) = 2$. Find $E(3X - 4)$.

2. **4 marks · Paper 2 style · calculator**

Given independent events A and B with $P(A) = 0.6$ and $P(B) = 0.4$, find $P(A \cup B)$.

3. **2 marks · Paper 2 style · calculator**

A basketball player scores 80% of free throws. She takes 10 free throws.

Find the probability that she scores at least 9. **[2]**

4. **2 marks · Paper 2 style · calculator**

$X \sim N(100,\ 15^2)$.

Find $P(|X - 100| < 20)$. **[2]**

5. **3 marks · Paper 1 style · no calculator**

Find the number of ways to select a team of 5 containing 3 girls and 2 boys, from a pool of 7 girls and 6 boys.

6. **3 marks · Paper 2 style · calculator**

70% of emails are genuine. A filter marks 5% of genuine emails and 90% of other emails as spam.

An email is marked as spam. Find the probability that it is genuine. **[3]**

7. **3 marks · Paper 1 style · no calculator**

A data set has $n = 10$, $\sum x = 85$ and standard deviation 2. A value of 8.5 is added.

Find the new mean and state, with a reason, whether the standard deviation increases or decreases. **[3]**

8. 3 marks · Paper 2 style · calculator

The regression line of y on x is $y = 0.85x + 12$ with $r = 0.91$, for x between 20 and 80.

Estimate y when $x = 50$, and comment on the reliability of the estimate. **[3]**

9. 8 marks · Paper 1 style · no calculator

A continuous random variable X has PDF $f(x) = \frac{3}{4}(2x - x^2)$ for $0 \leq x \leq 2$. Calculate $\mathrm{Var}(X)$.

10. 5 marks · Paper 1 style · no calculator

A continuous random variable X has pdf $f(x) = \dfrac{3}{8}x^2$ for $0 \leq x \leq 2$.

(a) Find $P(X > 1)$. **[2]**

(b) Find the median of X. **[3]**

Answers and worked solutions

Question 1 — 2 marks

Answer: 11

Use linearity of expectation: $E(3X-4)=3E(X)-4$ (the variance is not needed) **M1**

Substitute: $3(5)-4=11$ **A1**

Question 2 — 4 marks

Answer: 0.76

Use independence: $P(A\cap B)=P(A)P(B)=0.6\times 0.4$ **M1**

$=0.24$ **A1**

Use the addition rule: $P(A\cup B)=0.6+0.4-0.24$ **M1**

$=0.76$ **A1**

Question 3 — 2 marks

Answer: 0.376

$P(X\geq 9)$, $X\sim B(10,0.8)$ **M1**

0.376 **A1**

Question 4 — 2 marks

Answer: 0.818

$P(80<X<120)$ **M1**

0.818 **A1**

Question 5 — 3 marks

Answer: 525

Choose 3 of 7 girls and 2 of 6 boys, and multiply: $\binom{7}{3}\times\binom{6}{2}$ **M1**

$\binom{7}{3}=35$, $\binom{6}{2}=15$ **A1**

$35\times 15=525$ **A1**

Question 6 — 3 marks

Answer: 0.115

$P(S)=0.035+0.27=0.305$ **A1**

$\dfrac{0.035}{0.305}$ **M1**

0.115 **A1**

Question 7 — 3 marks

Answer: mean 8.5; it decreases

new mean $\dfrac{93.5}{11}=8.5$ **A1**

the new value equals the mean **R1**

so the standard deviation decreases **A1**

Question 8 — 3 marks

Answer: 54.5; reliable

$0.85(50)+12$ **M1**

54.5 **A1**

reliable: strong correlation and interpolation **R1**

Question 9 — 8 marks

Answer: 0.2

Find $E(X)$ by integrating $xf(x)$ over $0\leq x\leq 2$: $E(X)=\int_0^2 \frac{3}{4}(2x^2-x^3)\,dx$ **M1**

Integrate: $\frac{3}{4}\left[\frac{2x^3}{3}-\frac{x^4}{4}\right]_0^2=\frac{3}{4}\left(\frac{16}{3}-4\right)$ **A1**

Simplify: $E(X)=\frac{3}{4}\times\frac{4}{3}=1$ **A1**

Find $E(X^2)$ by integrating $x^2f(x)$: $E(X^2)=\int_0^2 \frac{3}{4}(2x^3-x^4)\,dx$ **M1**

Integrate: $\frac{3}{4}\left[\frac{x^4}{2}-\frac{x^5}{5}\right]_0^2=\frac{3}{4}\left(8-\frac{32}{5}\right)$ **A1**

Simplify: $E(X^2)=\frac{3}{4}\times\frac{8}{5}=\frac{6}{5}$ **A1**

Use $\mathrm{Var}(X)=E(X^2)-[E(X)]^2$:

$\mathrm{Var}(X)=\frac{6}{5}-1^2$ **M1**

$\mathrm{Var}(X)=\frac{1}{5}=0.2$ **A1**

Question 10 — 5 marks

Answer: (a) $\frac{7}{8}$; (b) $\sqrt[3]{4}$

(a)

$\int_1^2 \frac{3}{8}x^2\,dx$ **M1**

$\frac{7}{8}$ **A1**

(b)

$\frac{m^3}{8}=\frac{1}{2}$ **M1A1**

$m=\sqrt[3]{4}$ **A1**

Every solution is also on ibmathrevision.com/hlaa/probability-and-statistics-exam-practice
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