IB MATHS AA HL · UNIT 2

Polynomials and Rational Functions: exam practice

10 questions · 40 marks · worked solutions and markschemes

Name _____ Date _____ Score / 40

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/polynomials-and-rational-functions-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Let the polynomial be $P(x) = 2x^3 - x^2 + kx - 4$. When $P(x)$ is divided by $(x - 2)$, the remainder is 10. Find the value of the constant k.

2. **3 marks · Paper 1 style · no calculator**

Find the equation of the oblique asymptote of the graph of $y = \dfrac{2x^2 - x - 4}{x - 2}$.

3. **4 marks · Paper 2 style · calculator**

Solve $x^3 - 2x^2 - 5x + 6 \leq 0$.

Solve the inequality. **[4]**

4. **4 marks · Paper 1 style · no calculator**

Find the quotient and remainder when $2x^3 - 3x^2 + 4x - 5$ is divided by $x^2 + 1$.

Find the quotient and remainder. **[4]**

5. **3 marks · Paper 1 style · no calculator**

The graph of a cubic $y = f(x)$ touches the x-axis at $x = 2$, crosses it at $x = -1$ and passes through $(0, 8)$.

Find $f(x)$ in the form $a(x - p)^2(x - q)$. **[3]**

6. **2 marks · Paper 1 style · no calculator**

Consider the rational function $f(x) = \frac{3x-2}{x+4}$. Write down the equations of its vertical and horizontal asymptotes.

7. **4 marks · Paper 1 style · no calculator**

Let $p(x) = x^4 - 5x^2 + 4$.

(a) Factorise $p(x)$ fully. **[2]**

(b) Find the y-intercept, and state the behaviour of $p(x)$ as $x \to \pm\infty$. **[2]**

8. **5 marks · Paper 1 style · no calculator**

The polynomial $P(x) = x^3 + ax^2 + bx + 6$ has a factor of $(x - 2)$ and leaves a remainder of 12 when divided by $(x + 1)$. Find the exact values of a and b.

9. **6 marks · Paper 1 style · no calculator**

A polynomial $P(x) = x^3 + ax^2 + bx + 10$ has a factor $(x - 2)$. When $P(x)$ is divided by $(x + 1)$, the remainder is 12. Find the values of a and b.

10. **6 marks · Paper 1 style · no calculator**

A cubic equation $z^3 + pz^2 + qz + 15 = 0$, where p and q are real constants, has $1 + 2i$ as one of its roots. Find the exact values of p and q.

Answers and worked solutions

Question 1 — 3 marks

Answer: 1

1. Recognizes the remainder theorem by setting $P(2) = 10$ **(M1)**
   Correct substitution into the polynomial $2(2)^3 - (2)^2 + k(2) - 4 = 10$ **A1**
   Simplifies to $8 + 2k = 10$, which gives $k = 1$ **A1**

Question 2 — 3 marks

Answer: $y = 2x + 3$

$\frac{2x^2 - x - 4}{x - 2} = 2x + 3 + \frac{2}{x - 2}$ $M1$**A1**

$y = 2x + 3$ **A1**

Question 3 — 4 marks

Answer: $x \leq -2$ or $1 \leq x \leq 3$

$(x - 1)(x + 2)(x - 3) \leq 0$ $M1$**A1**

$x \leq -2$ **A1**

$1 \leq x \leq 3$ **A1**

Question 4 — 4 marks

Answer: quotient $2x - 3$, remainder $2x - 2$

attempt at long division **M1**

quotient $2x - 3$ **A1A1**

remainder $2x - 2$ **A1**

Question 5 — 3 marks

Answer:

$f(x) = 2(x - 2)^2(x + 1)$

$f(x) = a(x - 2)^2(x + 1)$ **A1**

$8 = a(4)(1)$ **M1**

$f(x) = 2(x - 2)^2(x + 1)$ **A1**

Question 6 — 2 marks

Answer: (Vertical asymptote: $x =) -4$; (Horizontal asymptote: $y =) 3$

Vertical asymptote where $x + 4 = 0$: $x = -4$ **A1**

Horizontal asymptote from ratio of leading coefficients: $y = 3$ **A1**

Question 7 — 4 marks

Answer: (a) $(x - 1)(x + 1)(x - 2)(x + 2)$; (b) $(0, 4)$; $p(x) \to +\infty$

(a)

$(x^2 - 1)(x^2 - 4)$ **M1**

$(x - 1)(x + 1)(x - 2)(x + 2)$ **A1**

(b)

$(0, 4)$ **A1**

$p(x) \to +\infty$ at both ends **A1**

Question 8 — 5 marks

Answer: (a) 0; (b) -7

Factor theorem:

$P(2) = 0 \implies 8 + 4a + 2b + 6 = 0$ **M1**

$2a + b = -7$ **A1**

Remainder theorem:

$P(-1) = 12 \implies -1 + a - b + 6 = 12$ **M1**

$a - b = 7$ **A1**

Add the equations: $3a = 0$, so $a = 0$ and $b = -7$ **A1**

Question 9 — 6 marks

Answer: (a) -2; (b) -5

Factor theorem: $P(2) = 0$ **M1**

$8 + 4a + 2b + 10 = 0 \Rightarrow 2a + b = -9$ **A1**

Remainder theorem:

$P(-1) = 12$ **M1**

$-1 + a - b + 10 = 12 \Rightarrow a - b = 3$ **A1**

Add the equations: $3a = -6$ **M1**

$a = -2$, then $-2 - b = 3 \Rightarrow b = -5$ **A1**

Question 10 — 6 marks

Answer: $(p)\ 1$; $(q)\ -1$

Real coefficients, so $1 - 2i$ is also a root; let the third root be r **R1**

Product of roots:

$(1 + 2i)(1 - 2i)r = -15$ **M1**

$5r = -15 \Rightarrow r = -3$ **A1**

Sum of roots:

$-p = (1 + 2i) + (1 - 2i) + (-3)$; sum of pairs:

$q = (1 + 2i)(1 - 2i) + r[(1 + 2i) + (1 - 2i)]$ **M1**

$-p = -1 \Rightarrow p = 1$ **A1**

$q = 5 - 6 = -1$ **A1**

Every solution is also on ibmathrevision.com/hlaa/polynomials-and-rational-functions-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.