Planes: exam practice

10 questions · 42 marks · worked solutions and markschemes

Name _____ Date _____ Score / 42

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/planes-exam-practice

1. **2 marks · Paper 1 style · no calculator**

Find the scalar (dot) product $\mathbf{a} \cdot \mathbf{b}$ where $\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$.

2. **3 marks · Paper 2 style · calculator**

Find the angle, in degrees, between the vectors $\mathbf{u} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = 3\mathbf{i} + 4\mathbf{k}$.

3. **3 marks · Paper 1 style · no calculator**

The line $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ meets the plane $x + 3y + z = 13$ at P.

Find the coordinates of P. **[3]**

4. 3 marks · Paper 1 style · no calculator

Find the distance from the point $A(3, 1, -2)$ to the plane $2x - 2y + z = 10$.

Find the distance. **[3]**

5. 4 marks · Paper 1 style · no calculator

Find the acute angle between the planes $2x - y + 2z = 5$ and $x + 2y + 2z = 1$.

Find the angle. **[4]**

6. 5 marks · Paper 2 style · calculator

Particle A starts at $(0, 2, 1)$ and travels with velocity $\mathbf{v}_A = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ m/s. Particle B starts at $(3, 0, 0)$ and travels with velocity $\mathbf{v}_B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ m/s. Find the exact distance between the two particles after $t = 3$ seconds.

7. 6 marks · Paper 2 style · calculator

Calculate the angle between the line $L: \mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ and the plane $\Pi: 3x - 4y = 10$. Give your answer in degrees, correct to one decimal place.

8. 5 marks · Paper 1 style · no calculator

The planes $\Pi_1: x + y - z = 2$ and $\Pi_2: 2x - y + z = 4$ meet in a line L.

Find a vector equation of L. **[5]**

9. 4 marks · Paper 1 style · no calculator

$ABCD$ is a parallelogram with $A(1, 0, 2)$, $B(4, 1, 5)$ and $C(6, -2, 3)$.

a. Find $\overrightarrow{AB}$ and $\overrightarrow{BC}$. **[2]**

b. Find the coordinates of D. **[2]**

10. **7 marks · Paper 2 style · calculator**

Two helicopters are flying in linear paths defined by position vectors (in km) with time t in minutes:

$$\mathbf{r}_A = \begin{pmatrix} 10 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \mathbf{r}_B = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Find the time t at which the two helicopters are closest to each other, and state this exact minimum distance.

Answers and worked solutions

Question 1

2 marks

Answer: -12

Multiply corresponding components and add:

$\mathbf{a}\cdot\mathbf{b} = (2)(1) + (-3)(4) + (1)(-2)$ M1

$= 2 - 12 - 2 = -12$ A1

Question 2

3 marks

Answer: $94.7°$

Write the vectors in component form: $\mathbf{u} = (1, 2, -1)$, $\mathbf{v} = (3, 0, 4)$.

Scalar product:

$\mathbf{u}\cdot\mathbf{v} = 3 + 0 - 4 = -1$ A1

Magnitudes:

$|\mathbf{u}| = \sqrt{1+4+1} = \sqrt{6}$,

$|\mathbf{v}| = \sqrt{9+0+16} = 5$.

Use $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|} = \dfrac{-1}{5\sqrt{6}}$ M1

$\theta = 94.7°$ (3 s.f.) A1

Question 3

3 marks

Answer: $\left(6,\ \dfrac{5}{2},\ -\dfrac{1}{2}\right)$

$(1 + 2t) + 3t + (2 - t) = 13$ M1

$t = \dfrac{5}{2}$ A1

$P\left(6, \dfrac{5}{2}, -\dfrac{1}{2}\right)$ A1

Question 4

3 marks

Answer: $\frac{8}{3}$

$\dfrac{|2(3) - 2(1) + (-2) - 10|}{\sqrt{4+4+1}}$ $M1$A1

$\dfrac{8}{3}$ A1

Question 5

4 marks

Answer: $63.6°$

$\mathbf{n}_1\cdot\mathbf{n}_2 = 2 - 2 + 4 = 4$ A1

$|\mathbf{n}_1| = |\mathbf{n}_2| = 3$ A1

$\cos\theta = \dfrac{4}{9}$ M1

$63.6°$ A1

Question 6

5 marks

Answer: $4\sqrt{2}$

Position of A at time t:

$\mathbf{r}_A = (0, 2, 1) + t(1, -1, 2)$; at $t = 3$: $\mathbf{r}_A = (3, -1, 7)$ M1A1

Position of B at $t = 3$:

$\mathbf{r}_B = (3, 0, 0) + 3(0, 1, 1) = (3, 3, 3)$ A1

Vector between them:

$\overrightarrow{AB} = \mathbf{r}_B - \mathbf{r}_A = (0, 4, -4)$, so distance $= \sqrt{0 + 16 + 16}$ M1

$= \sqrt{32} = 4\sqrt{2}\ m$ A1

Question 7

6 marks

Answer: $19.5°$

Direction of the line: $\mathbf{d} = (1, 2, 2)$; normal to the plane $3x - 4y = 10$:

$\mathbf{n} = (3, -4, 0)$ A1

Dot product:

$\mathbf{d}\cdot\mathbf{n} = (1)(3) + (2)(-4) + (2)(0) = -5$ M1A1

Magnitudes: $|\mathbf{d}| = \sqrt{1+4+4} = 3$; $|\mathbf{n}| = \sqrt{9+16} = 5$.

Line–plane angle uses sine (the angle with the normal is $90° - \theta$):

$\sin\theta = \dfrac{|\mathbf{d}\cdot\mathbf{n}|}{|\mathbf{d}||\mathbf{n}|} = \dfrac{5}{15} = \dfrac{1}{3}$ M1A1

$\theta = \arcsin\left(\frac{1}{3}\right) = 19.5°$ (1 d.p.) A1

Question 8

5 marks

Answer: $\mathbf{r} = \begin{pmatrix}2\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\1\\1\end{pmatrix}$

direction $\mathbf{n}_1 \times \mathbf{n}_2 = \begin{pmatrix}0\\-3\\-3\end{pmatrix}$, use $\begin{pmatrix}0\\1\\1\end{pmatrix}$ M1A1

adding: $3x = 6$, $x = 2$ A1

point $(2, 0, 0)$ A1

$\mathbf{r} = \begin{pmatrix}2\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}0\\1\\1\end{pmatrix}$ A1

Question 9

4 marks

Answer: (AB) $(3, 1, 3)$; (BC) $(2, -3, -2)$; (D) $(3, -3, 0)$

a. $\overrightarrow{AB} = \begin{pmatrix}3\\1\\3\end{pmatrix}$ A1

$\overrightarrow{BC} = \begin{pmatrix}2\\-3\\-2\end{pmatrix}$ A1

b. $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC}$ (M1)

$D(3, -3, 0)$ A1

Question 10

7 marks

Answer: (t (minutes) 3; (Minimum distance (km) $\sqrt{35}$

Displacement from A to B:

$\mathbf{r}_B - \mathbf{r}_A = \begin{pmatrix}-10\\5\\0\end{pmatrix} + t\begin{pmatrix}3\\0\\1\end{pmatrix} = \begin{pmatrix}3t-10\\5\\t\end{pmatrix}$

M1A1

Square of the distance:

$D^2 = (3t - 10)^2 + 25 + t^2$ M1

$D^2 = 10t^2 - 60t + 125$ A1

Minimise: $\dfrac{d(D^2)}{dt} = 20t - 60 = 0$

M1

$t = 3$ minutes A1

$D^2 = 90 - 180 + 125 = 35$, so the minimum distance is $\sqrt{35}$ km (≈ 5.92 km) A1

Every solution is also on ibmathrevision.com/hlaa/planes-exam-practice

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