Partial Fractions: exam practice

10 questions · 48 marks · worked solutions and markschemes

Name _____ Date _____ Score / 48

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/partial-fractions-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Express $\dfrac{x+11}{(x+3)(x-1)}$ in partial fractions.

Write it in partial fractions. **[3]**

2. **3 marks · Paper 1 style · no calculator**

Express $\dfrac{x-7}{(x-2)^2}$ in the form $\dfrac{A}{x-2}+\dfrac{B}{(x-2)^2}$.

Find A and B. **[3]**

3. **4 marks · Paper 1 style · no calculator**

Express $\dfrac{7x-4}{x^2-x-2}$ in partial fractions.

Write it in partial fractions. **[4]**

4. **6 marks · Paper 1 style · no calculator**

Evaluate exactly the definite integral $\int_2^4 \frac{4}{(x-1)(x+3)}\,dx$.

5. **5 marks · Paper 1 style · no calculator**

Express $\dfrac{3x^2+4x+5}{(x+1)^2(x-3)}$ in partial fractions.

Write it in partial fractions. **[5]**

6. **4 marks · Paper 1 style · no calculator**

Find the exact values of the constants A and B in the partial fraction decomposition $\frac{5x-3}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$.

7. **5 marks · Paper 1 style · no calculator**

Express $\dfrac{6}{(x+1)(x+4)}$ in partial fractions and hence find $\displaystyle\int_0^2 \frac{6}{(x+1)(x+4)}\,dx$ in the form $a\ln b$.

(a) Partial fractions. **[2]**

(b) The integral. **[3]**

8. **6 marks · Paper 1 style · no calculator**

Let $f(x) = \dfrac{4}{x^2-4}$.

(a) Express $f(x)$ in partial fractions. **[3]**

(b) Hence find $\displaystyle\int_3^6 f(x)\,dx$, giving your answer in the form $\ln k$. **[3]**

9. 6 marks · Paper 1 style · no calculator

Let $g(x) = \dfrac{2x^2 - x + 3}{x^2 - 1}$.

(a) Write $g(x)$ in the form $2 + \dfrac{A}{x-1} + \dfrac{B}{x+1}$. **[4]**

(b) Find $\displaystyle\int g(x)\,dx$. **[2]**

10. 6 marks · Paper 2 style · calculator

Let $u_r = \dfrac{2}{r(r+2)}$.

(a) Express u_r in partial fractions. **[2]**

(b) Hence find $\displaystyle\sum_{r=1}^{n} u_r$ in terms of n. **[4]**

Answers and worked solutions

Question 1

3 marks

Answer: $\frac{3}{x-1}-\frac{2}{x+3}$

$x+11=A(x-1)+B(x+3)$ M1

$A=-2$, $B=3$ A1

$\frac{3}{x-1}-\frac{2}{x+3}$ A1

Question 2

3 marks

Answer: $A=1$, $B=-5$

$x-7=A(x-2)+B$ M1

$A=1$ A1

$B=-5$ A1

Question 3

4 marks

Answer: $\frac{10}{3(x-2)}+\frac{11}{3(x+1)}$

$x^2-x-2=(x-2)(x+1)$ A1

$7x-4=A(x+1)+B(x-2)$ M1

$A=\frac{10}{3}$, $B=\frac{11}{3}$ A1

$\frac{10}{3(x-2)}+\frac{11}{3(x+1)}$ A1

Question 4

6 marks

Answer: $\ln(15/7)$

Set up partial fractions:

$\frac{4}{(x-1)(x+3)}=\frac{A}{x-1}+\frac{B}{x+3}\Rightarrow 4=A(x+3)+B(x-1)$ M1

$x=1$: $A=1$; $x=-3$: $B=-1$ A1

Rewrite the integral:

$\int_2^4\left(\frac{1}{x-1}-\frac{1}{x+3}\right)dx$ M1

Integrate:

$\left[\ln|x-1|-\ln|x+3|\right]_2^4=\left[\ln\left|\frac{x-1}{x+3}\right|\right]_2^4$ A1

Substitute limits: $\ln\left(\frac{3}{7}\right)-\ln\left(\frac{1}{5}\right)$ M1

$=\ln\left(\frac{15}{7}\right)$ A1

Question 5

5 marks

Answer:

$\frac{1}{4(x+1)}-\frac{1}{(x+1)^2}+\frac{11}{4(x-3)}$

$\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-3}$ M1

$x=3$: $44=16C$, $C=\frac{11}{4}$ A1

$x=-1$: $4=-4B$, $B=-1$ A1

compare x^2: $3=A+C$, $A=\frac{1}{4}$ A1

$\frac{1}{4(x+1)}-\frac{1}{(x+1)^2}+\frac{11}{4(x-3)}$ A1

Question 6

4 marks

Answer: (A) 2; (B) 3

Multiply through by $(x+1)(x-3)$:

$5x-3=A(x-3)+B(x+1)$ M1

Substitute $x=3$: $12=4B$ (M1)

$B=3$ A1

Substitute $x=-1$: $-8=-4A$, so

$A=2$ A1

Question 7

5 marks

Answer: (a) $\frac{2}{x+1}-\frac{2}{x+4}$; (b) $2\ln 2$

(a)

$\frac{2}{x+1}-\frac{2}{x+4}$ A1A1

(b)

$\left[2\ln(x+1)-2\ln(x+4)\right]_0^2$ A1

$2\ln 3-2\ln 6+2\ln 4$ M1

$2\ln 2$ A1

Question 8

6 marks

Answer: (a) $\frac{1}{x-2}-\frac{1}{x+2}$; (b) $\ln\frac{5}{2}$

(a)

$\frac{A}{x-2}+\frac{B}{x+2}$ M1

$A=1$, $B=-1$ A1A1

(b)

$\left[\ln|x-2|-\ln|x+2|\right]_3^6$ A1

$=\ln\frac{4}{8}-\ln\frac{1}{5}$ M1

$\ln\frac{5}{2}$ A1

Question 9

6 marks

Answer: (a) $2+\frac{2}{x-1}-\frac{3}{x+1}$; (b) $2x+2\ln|x-1|-3\ln|x+1|+c$

(a)

$g(x)=2+\frac{5-x}{x^2-1}$ $M1$A1

$A=2$, $B=-3$ A1A1

(b)

$2x+2\ln|x-1|-3\ln|x+1|+c$ A1A1

Question 10

6 marks

Answer: (a) $\frac{1}{r}-\frac{1}{r+2}$; (b) $\frac{3}{2}-\frac{1}{n+1}-\frac{1}{n+2}$

(a)

$\frac{1}{r}-\frac{1}{r+2}$ A2

(b)

terms cancel in pairs (method of differences) M1

$1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}$ A1A1

$=\frac{3}{2}-\frac{1}{n+1}-\frac{1}{n+2}$ A1

Every solution is also on ibmathrevision.com/hlaa/partial-fractions-exam-practice
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