IB MATHS AA HL · UNIT 2

Odd, Even, Inverse, Inequalities and Modulus: exam practice

10 questions · 39 marks · worked solutions and markschemes

Name _____ Date _____ Score / 39

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/odd-even-inverse-inequalities-and-modulus-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Solve $|2x - 3| < 5$.

Solve the inequality. **[3]**

2. **3 marks · Paper 1 style · no calculator**

Show that $f(x) = \dfrac{3x + 2}{x - 3}$, $x \neq 3$, is a self-inverse function.

Show that f is self-inverse. **[3]**

3. **3 marks · Paper 1 style · no calculator**

The function f is odd and $f(x) = x^2 - 2x$ for $x \geq 0$.

Find $f(x)$ for $x < 0$, and find $f(-3)$. **[3]**

4. **3 marks · Paper 1 style · no calculator**

Let $f(x) = e^{x-2} + 1$.

Find $f^{-1}(x)$ and state its domain. **[3]**

5. **4 marks · Paper 1 style · no calculator**

Evaluate exactly the definite integral $\int_{-2}^{2} (x^5 \cos(x) + x^2)\, dx$.

6. **4 marks · Paper 2 style · calculator**

Solve $\dfrac{x+4}{x-1} \geq 3$.

Solve the inequality. **[4]**

7. **4 marks · Paper 2 style · calculator**

Let $f(x) = |x^2 - 4x|$.

Solve $f(x) = 3$. **[4]**

8. **5 marks · Paper 1 style · no calculator**

Given the functions $f(x) = \frac{1}{x+2}$ for $x \neq -2$, and $g(x) = 3x - 1$. Find the explicit expression for the inverse composite function $(f \circ g)^{-1}(x)$.

9. **4 marks · Paper 1 style · no calculator**

Solve $|x + 1| = |3x - 5|$.

Find x. **[4]**

10. **6 marks · Paper 1 style · no calculator**

Solve the equation $|2x - 1| = |x^2 - 4|$ algebraically.

Answers and worked solutions

Question 1 — 3 marks

Answer: $-1 < x < 4$

$-5 < 2x - 3 < 5$ **M1**

$-1 < x < 4$ **A2**

Question 2 — 3 marks

Answer: a proof: the worked solution above is the full answer

$f(f(x)) = \dfrac{3\frac{3x+2}{x-3} + 2}{\frac{3x+2}{x-3} - 3}$ **M1**

$= \dfrac{11x}{11}$ **A1**

$= x$, so $f^{-1} = f$ **A1 AG**

Question 3 — 3 marks

Answer: $f(x) = -x^2 - 2x$; $f(-3) = -3$

$f(x) = -f(-x) = -(x^2 + 2x)$ $M1$**A1**

$f(-3) = -3$ **A1**

Question 4 — 3 marks

Answer: $f^{-1}(x) = 2 + \ln(x - 1)$, $x > 1$

$x - 2 = \ln(y - 1)$ **M1**

$f^{-1}(x) = 2 + \ln(x - 1)$ **A1**

$x > 1$ **A1**

Question 5 — 4 marks

Answer: $16/3$

Split:

$\int_{-2}^{2} x^5 \cos x \, dx + \int_{-2}^{2} x^2 \, dx$ **M1**

$(-x)^5 \cos(-x) = -x^5 \cos x$ is odd over a symmetric interval, so the first integral is 0 **R1**

$\int_{-2}^{2} x^2 \, dx = \left[\frac{x^3}{3}\right]_{-2}^{2}$ **A1**

$= \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$ **A1**

Question 6 — 4 marks

Answer: $1 < x \leq \dfrac{7}{2}$

$\dfrac{x + 4 - 3(x - 1)}{x - 1} \geq 0$ **M1**

$\dfrac{7 - 2x}{x - 1} \geq 0$ **A1**

critical values 1, $\dfrac{7}{2}$ **A1**

$1 < x \leq \dfrac{7}{2}$ **A1**

Question 7 — 4 marks

Answer: $x = 2 \pm \sqrt{7}$, $x = 1$, $x = 3$

$x^2 - 4x = 3$ or $x^2 - 4x = -3$ **M1**

$x = 2 \pm \sqrt{7}$ **A1**

$x = 1$, $x = 3$ **A1A1**

Question 8 — 5 marks

Answer: $(1 - x)/(3x)$

Form the composite:

$(f \circ g)(x) = \frac{1}{(3x-1)+2}$ **M1**

$(f \circ g)(x) = \frac{1}{3x+1}$ **A1**

Let $x = \frac{1}{3y+1}$ and solve for y **M1**

$3y + 1 = \frac{1}{x} \Rightarrow 3y = \frac{1-x}{x}$ **A1**

$(f \circ g)^{-1}(x) = \frac{1-x}{3x}$, $x \neq 0$ **A1**

Question 9 — 4 marks

Answer: $x = 1$, $x = 3$

$x + 1 = 3x - 5$ or $x + 1 = -(3x - 5)$ **M1**

$x = 3$ **A1**

$x = 1$ **A1**

both checked **R1**

Question 10 — 6 marks

Answer: $3, -1, -1 + \sqrt{6}, -1 - \sqrt{6}$

Split into two cases:

$2x - 1 = x^2 - 4$ or $2x - 1 = -(x^2 - 4)$ **M1**

Case 1: $x^2 - 2x - 3 = 0$ **A1**

$(x - 3)(x + 1) = 0 \implies x = 3$ or $x = -1$ **A1**

Case 2: $x^2 + 2x - 5 = 0$ **A1**

Use the quadratic formula:

$x = \frac{-2 \pm \sqrt{4+20}}{2} = \frac{-2 \pm \sqrt{24}}{2}$ **(M1)**

$x = -1 \pm \sqrt{6}$ (all four solutions satisfy the original equation) **A1**

Every solution is also on ibmathrevision.com/hlaa/odd-even-inverse-inequalities-and-modulus-exam-practice
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