IB MATHS AA HL · UNIT 1

Number & Algebra: exam practice

10 questions · 44 marks · worked solutions and markschemes

Name _____ Date _____ Score / 44

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/number-and-algebra-exam-practice

1. **4 marks · Paper 1 style · no calculator**

Find the coefficient of x^2 in the binomial expansion of $\left(2x - \frac{1}{x}\right)^4$.

2. **3 marks · Paper 1 style · no calculator**

Find the value of $\sum_{r=1}^{20}(3r - 2)$.

Find the sum. **[3]**

3. **3 marks · Paper 2 style · calculator**

Solve the system $x + y + z = 6$, $2x - y + 3z = 9$, $x + 2y - z = 2$.

Find x, y and z. **[3]**

4. **4 marks · Paper 1 style · no calculator**

Solve $e^{2x} - 5e^{x} + 6 = 0$, giving exact answers.

Find x. **[4]**

5. **4 marks · Paper 1 style · no calculator**

Express $\dfrac{5x + 1}{(x - 1)(x + 2)}$ in partial fractions.

Write it in partial fractions. **[4]**

6. **5 marks · Paper 1 style · no calculator**

Find the real numbers a and b such that $(a + bi)^2 = -5 + 12i$.

Find a and b. **[5]**

7. **6 marks · Paper 1 style · no calculator**

Determine the value of k for which the system of equations has infinitely many solutions:

$$x + y + z = 2$$
$$2x + 3y + 2z = 5$$
$$3x + 4y + kz = 7$$

8. **6 marks · Paper 1 style · no calculator**

Prove by mathematical induction that $7^n - 1$ is divisible by 6 for all $n \in \mathbb{Z}^+$.

Write the proof. **[6]**

9. **4 marks · Paper 1 style · no calculator**

Given the complex number $z = 3 - 3i$, find the exact modulus $|z|$ and argument $\arg(z)$ in radians.

10. **5 marks · Paper 1 style · no calculator**

An infinite geometric series has first term 24 and sum to infinity 96.

(a) Find the common ratio. **[2]**

(b) Find the least n for which the sum of the first n terms exceeds 95. **[3]**

Answers and worked solutions

Question 1 — 4 marks

Answer: -32

Write the general term:

$\binom{4}{r}(2x)^{4-r}\left(-\frac{1}{x}\right)^r = \binom{4}{r}2^{4-r}(-1)^r x^{4-2r}$ **M1**

Set the power of x equal to 2:

$4-2r=2 \implies r=1$ **A1**

Substitute $r=1$ into the coefficient:

$\binom{4}{1}2^3(-1)^1 = 4\times 8\times(-1)$ **M1**

Coefficient of x^2 is -32 **A1**

Question 2 — 3 marks

Answer: 590

arithmetic, $u_1=1$, $u_{20}=58$ **A1**

$\frac{20}{2}(1+58)$ **M1**

590 **A1**

Question 3 — 3 marks

Answer: $x=1$, $y=2$, $z=3$

attempt to solve (GDC or elimination) **M1**

$x=1$, $y=2$, $z=3$ **A2**

Question 4 — 4 marks

Answer: $x=\ln 2$, $x=\ln 3$

$(e^x-2)(e^x-3)=0$ $M1$**A1**

$x=\ln 2$, $x=\ln 3$ **A1A1**

Question 5 — 4 marks

Answer: $\frac{2}{x-1}+\frac{3}{x+2}$

$5x+1=A(x+2)+B(x-1)$ **M1**

$x=1$: $A=2$ **A1**

$x=-2$: $B=3$ **A1**

$\frac{2}{x-1}+\frac{3}{x+2}$ **A1**

Question 6 — 5 marks

Answer: $a=2,\ b=3$ or $a=-2,\ b=-3$

$a^2-b^2=-5$, $2ab=12$ $M1$ **A1**

$a^2-\frac{36}{a^2}=-5$ **M1**

$a=2, b=3$ or $a=-2, b=-3$ **A1A1**

Question 7 — 6 marks

Answer: 3

Row reduce: $R_2\to R_2-2R_1$ gives $y=1$ **M1 A1**

$R_3\to R_3-3R_1$ gives $y+(k-3)z=1$ **A1**

Substitute $y=1$:

$1+(k-3)z=1 \implies (k-3)z=0$ **M1**

If $k\neq 3$ then $z=0$ (unique solution); if $k=3$ the equation is $0=0$, so z is free and there are infinitely many solutions **R1**

$k=3$ **A1**

Question 8 — 6 marks

Answer: a proof: the worked solution above is the full answer

$n=1$: $7-1=6$, divisible by 6 **A1**

assume $7^k-1=6m$ for some $m\in\mathbb{Z}$ **M1**

$7^{k+1}-1=7\cdot 7^k-1=7(6m+1)-1$ **M1**

$=42m+6=6(7m+1)$ **A1**

divisible by 6 **A1**

true for $n=1$, and true for $n=k$ implies true for $n=k+1$, so true for all $n\in\mathbb{Z}^+$ **R1**

Question 9 — 4 marks

Answer: $(|z|)\ 3\sqrt{2}$; $(\arg z)\ -\pi/4$

Calculate the modulus:

$|z|=\sqrt{3^2+(-3)^2}=\sqrt{18}$ **M1**

$|z|=3\sqrt{2}$ **A1**

z is in the fourth quadrant, so

$\arg z=\arctan\left(\frac{-3}{3}\right)$ **M1**

$\arg z=-\frac{\pi}{4}$ **A1**

Question 10 — 5 marks

Answer: (a) $\frac{3}{4}$; (b) 16

(a)

$\frac{24}{1-r}=96$ **M1**

$r=\frac{3}{4}$ **A1**

(b)

$96\left(1-0.75^n\right)>95$ **M1**

$n>15.9$ **A1**

$n=16$ **A1**

Every solution is also on ibmathrevision.com/hlaa/number-and-algebra-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.