IB MATHS AA HL · UNIT 5

Limits and Rates of Change: exam practice

10 questions · 30 marks · worked solutions and markschemes

Name _____ Date _____ Score / 30

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/limits-and-rates-of-change-exam-practice

1. **2 marks · Paper 1 style · no calculator**

Find $\lim_{x \to \infty} \dfrac{3x^2 - 2x}{5x^2 + 1}$.

Find the limit. **[2]**

2. **2 marks · Paper 1 style · no calculator**

Find $\lim_{x \to 0} \dfrac{\sin 5x}{x}$.

Find the limit. **[2]**

3. **2 marks · Paper 1 style · no calculator**

The population of a colony is $P(t) = \dfrac{500}{1 + 4e^{-0.5t}}$.

Find the initial population and the limiting population. **[2]**

4. 3 marks · Paper 1 style · no calculator

Find $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4}$.

Find the limit. **[3]**

5. 3 marks · Paper 1 style · no calculator

The volume of water in a tank is $V(t) = 200 + 30t - t^2$ litres after t minutes.

Find the rate of change of volume at $t = 10$, and the time when the volume stops increasing. **[3]**

6. 3 marks · Paper 1 style · no calculator

Find $\lim_{x \to \infty} \left(2 + \dfrac{3}{e^x}\right)$ and $\lim_{x \to 0^+} x \ln x$.

Find both limits. **[3]**

7. 3 marks · Paper 1 style · no calculator

The gradient of the chord from $(2, f(2))$ to $(2 + h, f(2 + h))$ on $f(x) = x^2 + 3x$ is $7 + h$.

Show this, and hence write down $f'(2)$. **[3]**

8. **4 marks · Paper 1 style · no calculator**

Use first principles to find the derivative of $f(x) = \frac{1}{x}$.

Find $f'(x)$. **[4]**

9. **4 marks · Paper 1 style · no calculator**

Use first principles to find $f'(x)$ for $f(x) = x^3$.

Find $f'(x)$. **[4]**

10. **4 marks · Paper 1 style · no calculator**

Use first principles to find the derivative of $f(x) = \sqrt{x}$.

Find $f'(x)$. **[4]**

Answers and worked solutions

Question 1 2 marks

Answer: $\frac{3}{5}$

divide by x^2 **M1**

$\frac{3}{5}$ **A1**

Question 2 2 marks

Answer: 5

$5 \cdot \frac{\sin 5x}{5x}$ **M1**

5 **A1**

Question 3 2 marks

Answer: 100; 500

$P(0) = 100$ **A1**

$\lim_{t \to \infty} P = 500$ **A1**

Question 4 3 marks

Answer: $\frac{1}{4}$

$x - 4 = (\sqrt{x} - 2)(\sqrt{x} + 2)$ **M1**

$\frac{1}{\sqrt{x} + 2}$ **A1**

$\frac{1}{4}$ **A1**

Question 5 3 marks

Answer: 10 litres/min; $t = 15$

$V'(t) = 30 - 2t$ **A1**

$V'(10) = 10$ litres per minute **A1**

$t = 15$ min **A1**

Question 6 3 marks

Answer: 2; 0

2 **A1**

$\frac{\ln x}{1/x}$, l'Hôpital: $\frac{1/x}{-1/x^2} = -x$ **M1**

0 **A1**

Question 7 3 marks

Answer: 7

$\frac{(2+h)^2 + 3(2+h) - 10}{h}$ **M1**

$= \frac{7h + h^2}{h} = 7 + h$ **A1 AG**

$f'(2) = 7$ **A1**

Question 8 4 marks

Answer: $-\frac{1}{x^2}$

$\lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$ **M1**

$= \lim_{h \to 0} \frac{-h}{hx(x+h)}$ **A1**

$= \lim_{h \to 0} \frac{-1}{x(x+h)}$ **A1**

$-\frac{1}{x^2}$ **A1**

Question 9 4 marks

Answer: $3x^2$

$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$ **M1**

$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$ **A1**

$= \lim_{h \to 0} (3x^2 + 3xh + h^2)$ **A1**

$3x^2$ **A1**

Question 10 4 marks

Answer: $\frac{1}{2\sqrt{x}}$

$\lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$ **M1**

multiply by $\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$ **M1**

$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}$ **A1**

$\frac{1}{2\sqrt{x}}$ **A1**

Every solution is also on ibmathrevision.com/hlaa/limits-and-rates-of-change-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.