IB MATHS AA HL · UNIT 5

Integration: exam practice

10 questions · 28 marks · worked solutions and markschemes

Name _____ Date _____ Score / 28

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/integration-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Find $\int \left(4x^3 - \frac{3}{x} + e^{2x}\right) dx$.

Find the integral. **[3]**

2. **2 marks · Paper 1 style · no calculator**

Find $\int \frac{1}{3x-2}\, dx$, for $x > \frac{2}{3}$.

Find the integral. **[2]**

3. **2 marks · Paper 1 style · no calculator**

Find $\int (2x+1)^5\, dx$.

Find the integral. **[2]**

4. **3 marks · Paper 1 style · no calculator**

A curve has gradient $\frac{dy}{dx} = 6x^2 - 4x$ and passes through $(1, 5)$.

Find the equation of the curve. **[3]**

5. **3 marks · Paper 1 style · no calculator**

Find $\int \left(\sec^2 x + \sin 3x\right) dx$.

Find the integral. **[3]**

6. **3 marks · Paper 1 style · no calculator**

Find $\int x(x^2 + 1)^4 \, dx$.

Find the integral. **[3]**

7. **3 marks · Paper 1 style · no calculator**

Find $\int \cos^2 x \, dx$.

Find the integral. **[3]**

8. **3 marks · Paper 1 style · no calculator**

Given $f'(x) = \dfrac{2}{\sqrt{x}} - 1$ and $f(4) = 3$, find $f(x)$.

Find $f(x)$. **[3]**

9. **4 marks · Paper 1 style · no calculator**

Find $\displaystyle\int \frac{(x+2)^2}{\sqrt{x}}\,dx$.

Find the integral. **[4]**

10. **2 marks · Paper 1 style · no calculator**

Find $\displaystyle\int \frac{e^x}{e^x+4}\,dx$.

Find the integral. **[2]**

Answers and worked solutions

Question 1 — 3 marks

Answer:

$x^4 - 3\ln|x| + \frac{1}{2}e^{2x} + c$

x^4 A1

$-3\ln|x|$ A1

$+\frac{1}{2}e^{2x} + c$ A1

Question 2 — 2 marks

Answer: $\frac{1}{3}\ln(3x-2) + c$

$\frac{1}{3}\ln(3x-2)$ A1

$+c$ A1

Question 3 — 2 marks

Answer: $\frac{(2x+1)^6}{12} + c$

$\frac{(2x+1)^6}{12}$ A1

$+c$ A1

Question 4 — 3 marks

Answer: $y = 2x^3 - 2x^2 + 5$

$y = 2x^3 - 2x^2 + c$ A1

$5 = 2 - 2 + c$ M1

$y = 2x^3 - 2x^2 + 5$ A1

Question 5 — 3 marks

Answer: $\tan x - \frac{1}{3}\cos 3x + c$

$\tan x$ A1

$-\frac{1}{3}\cos 3x$ A1

$+c$ A1

Question 6 — 3 marks

Answer: $\frac{(x^2+1)^5}{10} + c$

$u = x^2 + 1,\ du = 2x\,dx$ M1

$\frac{1}{2}\int u^4\,du$ A1

$\frac{(x^2+1)^5}{10} + c$ A1

Question 7 — 3 marks

Answer: $\frac{x}{2} + \frac{\sin 2x}{4} + c$

$\cos^2 x = \frac{1+\cos 2x}{2}$ M1

$\frac{x}{2} + \frac{\sin 2x}{4}$ A1

$+c$ A1

Question 8 — 3 marks

Answer: $f(x) = 4\sqrt{x} - x - 1$

$f(x) = 4\sqrt{x} - x + c$ A1

$3 = 8 - 4 + c$ M1

$f(x) = 4\sqrt{x} - x - 1$ A1

Question 9 — 4 marks

Answer:

$\frac{2}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 8\sqrt{x} + c$

$\int \left(x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}\right)dx$ $M1$A1

$\frac{2}{5}x^{\frac{5}{2}} + \frac{8}{3}x^{\frac{3}{2}} + 8x^{\frac{1}{2}}$ A1

$+c$ A1

Question 10 — 2 marks

Answer: $\ln(e^x + 4) + c$

form $\frac{f'}{f}$ M1

$\ln(e^x + 4) + c$ A1

Every solution is also on ibmathrevision.com/hlaa/integration-exam-practice
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