IB MATHS AA HL · UNIT 5

Graphical Behaviour and The Second Derivative: exam practice

10 questions · 36 marks · worked solutions and markschemes

Name _____ Date _____ Score / 36

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/graphical-behaviour-and-the-second-derivative-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Let $f(x) = x^3 - 3x^2 - 9x + 5$.

Find the coordinates of the point of inflexion. **[3]**

2. **3 marks · Paper 1 style · no calculator**

Let $f(x) = 2x^3 + ax^2 + 5$. The graph of f has a point of inflexion at $x = 2$.

Find a. **[3]**

3. **3 marks · Paper 1 style · no calculator**

Let $f(x) = 2x^3 - 3x^2 - 12x + 4$.

Find the intervals on which f is decreasing. **[3]**

4. **4 marks · Paper 1 style · no calculator**

Let $f(x) = x^4 - 6x^2$.

Find the x-coordinates of the points of inflexion and justify that they are points of inflexion. **[4]**

5. **4 marks · Paper 1 style · no calculator**

Let $f(x) = x^3 - 6x^2 + 12x$.

Show that the graph has a stationary point of inflexion and find its coordinates. **[4]**

6. **4 marks · Paper 1 style · no calculator**

Let $f(x) = \ln(x^2 + 1)$.

Find the intervals on which the graph of f is concave up. **[4]**

7. **4 marks · Paper 1 style · no calculator**

The function f has $f'(x) = (x - 1)^2(x + 3)$.

Find the x-coordinate of the local minimum and the x-coordinate of the stationary point of inflexion. **[4]**

8. **4 marks · Paper 2 style · calculator**

Let $f(x) = \dfrac{1}{1+x^2}$.

Find the exact x-coordinates of the points of inflexion. **[4]**

9. **3 marks · Paper 1 style · no calculator**

Let $f(x) = \sin x + \cos x$ for $0 \leq x \leq 2\pi$.

Find the x-coordinates of the points of inflexion. **[3]**

10. **4 marks · Paper 1 style · no calculator**

Let $f(x) = xe^{-x}$.

Find the coordinates of the point of inflexion. **[4]**

Answers and worked solutions

Question 1 — 3 marks

Answer: $(1, -6)$

$f''(x) = 6x - 6 = 0$ **M1**

$x = 1$ **A1**

$(1, -6)$ **A1**

Question 2 — 3 marks

Answer: -12

$f''(x) = 12x + 2a$ **A1**

$24 + 2a = 0$ **M1**

$a = -12$ **A1**

Question 3 — 3 marks

Answer: $-1 < x < 2$

$f'(x) = 6(x-2)(x+1)$ **A1**

$f'(x) < 0$ **M1**

$-1 < x < 2$ **A1**

Question 4 — 4 marks

Answer: $x = \pm 1$

$f''(x) = 12x^2 - 12$ **A1**

$x = \pm 1$ **A1**

f'' changes sign at each **R1**

so both are points of inflexion **A1**

Question 5 — 4 marks

Answer: $(2, 8)$

$f'(x) = 3(x-2)^2$ **A1**

$f'(2) = 0$ and $f' \geq 0$ either side **R1**

$f''(2) = 0$ **A1**

$(2, 8)$ **A1**

Question 6 — 4 marks

Answer: $-1 < x < 1$

$f'(x) = \dfrac{2x}{x^2+1}$ **A1**

$f''(x) = \dfrac{2(1-x^2)}{(x^2+1)^2}$ **A1**

$f''(x) > 0$ **M1**

$-1 < x < 1$ **A1**

Question 7 — 4 marks

Answer: minimum at $x = -3$; inflexion at $x = 1$

$f'(x) = 0$ at $x = 1$, $x = -3$ **A1**

sign of f' changes $-$ to $+$ at $x = -3$: minimum $R1$**A1**

no sign change at $x = 1$: inflexion **A1**

Question 8 — 4 marks

Answer: $x = \pm \dfrac{1}{\sqrt{3}}$

$f'(x) = -\dfrac{2x}{(1+x^2)^2}$ **A1**

$f''(x) = \dfrac{6x^2 - 2}{(1+x^2)^3}$ **A1**

$6x^2 - 2 = 0$ **M1**

$x = \pm \dfrac{1}{\sqrt{3}}$ **A1**

Question 9 — 3 marks

Answer: $x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$

$f''(x) = -\sin x - \cos x$ **A1**

$\tan x = -1$ **M1**

$x = \dfrac{3\pi}{4}, \dfrac{7\pi}{4}$ **A1**

Question 10 — 4 marks

Answer: $\left(2,\ 2e^{-2}\right)$

$f'(x) = (1-x)e^{-x}$ **A1**

$f''(x) = (x-2)e^{-x}$ **A1**

$x = 2$ **A1**

$\left(2,\ \dfrac{2}{e^2}\right)$ **A1**

Every solution is also on ibmathrevision.com/hlaa/graphical-behaviour-and-the-second-derivative-exam-practice

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