IB MATHS AA HL · UNIT 3

Geometry & Trigonometry: exam practice

10 questions · 43 marks · worked solutions and markschemes

Name _____ Date _____ Score / 43

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/geometry-and-trigonometry-exam-practice

1. **2 marks · Paper 1 style · no calculator**

Find the exact value of $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$ in radians.

2. **4 marks · Paper 2 style · calculator**

Solve the equation $\sec^2 x = 4\tan x - 3$ for $0 \leq x \leq 2\pi$. Give your answers correct to 3 significant figures.

3. **4 marks · Paper 1 style · no calculator**

Find the area of the triangle with vertices $P(1, 0, 2)$, $Q(3, 1, 0)$ and $R(0, 2, 1)$.

Find the area. **[4]**

4. **5 marks · Paper 1 style · no calculator**

The points $A(1, 0, 2)$, $B(3, 1, 0)$ and $C(0, 2, 1)$ are the vertices of a triangle. Use the vector product to find the exact area of triangle ABC.

5. **5 marks · Paper 1 style · no calculator**

Given that $\sin\theta = \frac{5}{13}$ and $\frac{\pi}{2} < \theta < \pi$, find the exact values of $\cos 2\theta$ and $\sin 2\theta$.

Find $\cos 2\theta$ and $\sin 2\theta$. **[5]**

6. **5 marks · Paper 1 style · no calculator**

Find the Cartesian equation of the plane through $A(1, 2, 0)$, $B(3, 0, 1)$ and $C(0, 1, 2)$.

Find the equation. **[5]**

7. 5 marks · Paper 2 style · calculator

Solve $3\cos^2 x + 5\sin x = 1$ for $0 \leq x \leq 2\pi$.

Find x. **[5]**

8. 3 marks · Paper 1 style · no calculator

Let $\mathbf{u} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$.

a. Find $\mathbf{u} \cdot \mathbf{w}$. **[1]**

b. Find the exact value of the cosine of the angle between $\mathbf{u}$ and $\mathbf{w}$. **[2]**

9. 5 marks · Paper 1 style · no calculator

Let $\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} k \\ 4 \\ -2 \end{pmatrix}$.

(a) Find k given that $\mathbf{a}$ and $\mathbf{b}$ are perpendicular. **[3]**

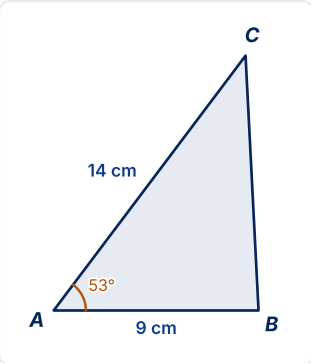
(b) For this k, find $|\mathbf{b}|$. **[2]**

10. **5 marks · Paper 2 style · calculator**

In triangle ABC, $AB = 9$ cm, $AC = 14$ cm and $B\hat{A}C = 53°$.

(a) Find BC. **[3]**

(b) Find the area of the triangle. **[2]**



AB = 9 cm, AC = 14 cm, angle A = 53°

Answers and worked solutions

Question 1 — 2 marks

Answer: $-\pi/3$

Recall the range of $\arcsin$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ **M1**

$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ **A1**

Question 2 — 4 marks

Answer: $1.11, 4.25$

Use $\sec^2 x = 1 + \tan^2 x$:

$1 + \tan^2 x = 4\tan x - 3$ **M1**

$\tan^2 x - 4\tan x + 4 = 0$ **A1**

$(\tan x - 2)^2 = 0 \implies \tan x = 2$ **A1**

$x = \arctan 2 \approx 1.11$ and $x = \pi + \arctan 2 \approx 4.25$ **A1**

Question 3 — 4 marks

Answer: $\dfrac{5\sqrt{2}}{2}$

$\overrightarrow{PQ} = \begin{pmatrix}2\\1\\-2\end{pmatrix}, \overrightarrow{PR} = \begin{pmatrix}-1\\2\\-1\end{pmatrix}$ **A1**

cross product $\begin{pmatrix}3\\4\\5\end{pmatrix}$ $M1A1$

area $= \frac{1}{2}\sqrt{50} = \frac{5\sqrt{2}}{2}$ **A1**

Question 4 — 5 marks

Answer: $5\sqrt{2}/2$

$\overrightarrow{AB} = \begin{pmatrix}2\\1\\-2\end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix}-1\\2\\-1\end{pmatrix}$ **A1**

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix}(1)(-1)-(-2)(2)\\(-2)(-1)-(2)(-1)\\(2)(2)-(1)(-1)\end{pmatrix}$ **M1**

$= \begin{pmatrix}3\\4\\5\end{pmatrix}$ **A1**

Area $= \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \frac{1}{2}\sqrt{9+16+25}$ **M1**

$= \frac{\sqrt{50}}{2} = \frac{5\sqrt{2}}{2}$ **A1**

Question 5 — 5 marks

Answer: $\cos 2\theta = \dfrac{119}{169}$, $\sin 2\theta = -\dfrac{120}{169}$

$\cos\theta = -\frac{12}{13}$ **A1**

$\cos 2\theta = 1 - 2\left(\frac{25}{169}\right)$ **M1**

$= \frac{119}{169}$ **A1**

$\sin 2\theta = 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right)$ **M1**

$= -\frac{120}{169}$ **A1**

Question 6 — 5 marks

Answer: $3x + 5y + 4z = 13$

$\overrightarrow{AB} = \begin{pmatrix}2\\-2\\1\end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix}-1\\-1\\2\end{pmatrix}$ **A1**

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix}-3\\-5\\-4\end{pmatrix}$ $M1A1$

$3x + 5y + 4z = d$, with $d = 3 + 10 = 13$ **M1**

$3x + 5y + 4z = 13$ **A1**

Question 7 — 5 marks

Answer: $x = 3.48$, $x = 5.94$

$3(1 - \sin^2 x) + 5\sin x - 1 = 0$ **M1**

$3\sin^2 x - 5\sin x - 2 = 0$ **A1**

$(3\sin x + 1)(\sin x - 2) = 0$,

$\sin x = -\frac{1}{3}$ **A1**

$x = 3.48, x = 5.94$ **A1A1**

Question 8 — 3 marks

Answer: (a) -5; (b) $-1/3$

a. $3 + 0 - 8 = -5$ **A1**

b. $\cos\theta = \dfrac{-5}{3\times 5}$ **M1**

$= -\dfrac{1}{3}$ **A1**

Question 9 — 5 marks

Answer: (a) 5; (b) $3\sqrt{5}$

(a)

$\mathbf{a}\cdot\mathbf{b} = 2k - 4 - 6$ $M1A1$

$k = 5$ **A1**

(b)

$\sqrt{25+16+4}$ **M1**

$3\sqrt{5}$ **A1**

Question 10 — 5 marks

Answer: (a) 11.2 cm; (b) $50.3\ \text{cm}^2$

(a)

$BC^2 = 9^2 + 14^2 - 2(9)(14)\cos 53°$ $M1A1$

$BC = 11.2$ cm **A1**

(b)

$\frac{1}{2}(9)(14)\sin 53°$ **M1**

$50.3\ \text{cm}^2$ **A1**

Every solution is also on ibmathrevision.com/hlaa/geometry-and-trigonometry-exam-practice
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