Functions: exam practice

10 questions · 42 marks · worked solutions and markschemes

Name _____ Date _____ Score / 42

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/functions-exam-practice

1. **4 marks · Paper 1 style · no calculator**

A cubic polynomial $P(x) = x^3 - 4x^2 + kx - 6$ has a root at $x = 2$. Find the value of k.

2. **3 marks · Paper 1 style · no calculator**

The graph of $y = x^2$ is stretched vertically by a factor of 3, then translated by $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$.

Find the equation of the new graph in the form $y = ax^2 + bx + c$. **[3]**

3. **3 marks · Paper 1 style · no calculator**

The equation $kx^2 - 4x + k = 0$ has two equal real roots.

Find the possible values of k. **[3]**

4. **4 marks · Paper 1 style · no calculator**

Let $f(x) = \dfrac{2x - 1}{x + 3}$.

Write down the equations of the asymptotes and the coordinates of the intercepts of the graph of f. **[4]**

5. **4 marks · Paper 2 style · calculator**

Solve the inequality $x^3 - 4x > 2x^2 - 8$, giving your answer in interval form.

Solve the inequality. **[4]**

6. **4 marks · Paper 1 style · no calculator**

Determine whether each function is odd, even or neither: $f(x) = x^3 \cos x$ and $g(x) = x^2 + x$.

Classify f and g. **[4]**

7. **5 marks · Paper 1 style · no calculator**

When $p(x) = x^3 + ax^2 + bx - 6$ is divided by $(x - 1)$ the remainder is -4, and $(x + 3)$ is a factor of $p(x)$.

Find a and b. **[5]**

8. **4 marks · Paper 2 style · calculator**

Determine the oblique asymptote of the rational function $y = \frac{x^2 - x - 2}{x - 3}$ using polynomial division.

9. **8 marks · Paper 1 style · no calculator**

Solve the equation $|x^2 - 1| = x + 5$ algebraically.

10. **3 marks · Paper 1 style · no calculator**

Let $f(x) = \sqrt{x-1}$, $x \geq 1$, and $g(x) = x^2 + 2$.

(a) Find $(f \circ g)(x)$. **[2]**

(b) Find the range of $f \circ g$. **[1]**

Answers and worked solutions

Question 1 — 4 marks

Answer: 7

Apply the factor theorem:

$P(2) = 0$ **M1**

Substitute $x = 2$:

$8 - 16 + 2k - 6 = 0$ **A1**

Simplify: $2k - 14 = 0$ **A1**

$k = 7$ **A1**

Question 2 — 3 marks

Answer: $y = 3x^2 + 12x + 17$

$y = 3x^2$ **A1**

$y = 3(x + 2)^2 + 5$ **A1**

$y = 3x^2 + 12x + 17$ **A1**

Question 3 — 3 marks

Answer: $k = \pm 2$

$\Delta = 16 - 4k^2 = 0$ $M1$**A1**

$k = \pm 2$ **A1**

Question 4 — 4 marks

Answer: $x = -3$, $y = 2$, $\left(\frac{1}{2}, 0\right)$, $\left(0, -\frac{1}{3}\right)$

$x = -3$ **A1**

$y = 2$ **A1**

$\left(\frac{1}{2}, 0\right)$ **A1**

$\left(0, -\frac{1}{3}\right)$ **A1**

Question 5 — 4 marks

Answer: $-2 < x < 2$ or $x > 2$

$x^3 - 2x^2 - 4x + 8 > 0$ **M1**

$(x - 2)^2(x + 2) > 0$ **A1**

$x > -2$, $x \neq 2$ **A1A1**

Question 6 — 4 marks

Answer: f odd, g neither

$f(-x) = -x^3 \cos x = -f(x)$ **M1**

f is odd **A1**

$g(-x) = x^2 - x$, not $\pm g(x)$ **M1**

g is neither **A1**

Question 7 — 5 marks

Answer: $a = 3$, $b = -2$

$p(1) = 1 + a + b - 6 = -4$ $M1$**A1**

$p(-3) = -27 + 9a - 3b - 6 = 0$ **A1**

attempt to solve **M1**

$a = 3$, $b = -2$ **A1**

Question 8 — 4 marks

Answer: $y = x + 2$

Divide $x^2 - x - 2$ by $x - 3$: **M1**

$\frac{x^2 - x - 2}{x - 3} = x + 2 + \frac{4}{x - 3}$ **A1**

As $x \to \pm\infty$, $\frac{4}{x-3} \to 0$ **R1**

Oblique asymptote: $y = x + 2$ **A1**

Question 9 — 8 marks

Answer: $-2, 3$

Case 1 ($x^2 - 1 \geq 0$, i.e. $x \leq -1$ or $x \geq 1$):

$x^2 - 1 = x + 5$ **M1**

$x^2 - x - 6 = 0$ **A1**

$(x - 3)(x + 2) = 0 \implies x = 3$ or $x = -2$ **A1**

Both satisfy $x^2 - 1 \geq 0$ (and give $x + 5 > 0$), so both are valid **R1**

Case 2 ($-1 < x < 1$):

$-(x^2 - 1) = x + 5$ **M1**

$x^2 + x + 4 = 0$ **A1**

Discriminant $\Delta = 1^2 - 4(1)(4) = -15 < 0$, so there are no real solutions in this case **R1**

Solutions: $x = -2, 3$ **A1**

Question 10 — 3 marks

Answer: (a) $\sqrt{x^2 + 1}$; (b) $(f \circ g)(x) \geq 1$

(a)

$\sqrt{x^2 + 2 - 1}$ **M1**

$\sqrt{x^2 + 1}$ **A1**

(b)

$(f \circ g)(x) \geq 1$ **A1**

Every solution is also on ibmathrevision.com/hlaa/functions-exam-practice
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