IB MATHS AA HL · UNIT 2

# Function fundamentals: exam practice

10 questions · 40 marks · worked solutions and markschemes

Name _____ Date _____ Score / 40

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/function-fundamentals-exam-practice

**1.** **2 marks · Paper 1 style · no calculator**

Let $f(x) = \dfrac{1}{x^2 - 9}$.

Write down the largest possible domain of $f$. **[2]**

**2.** **3 marks · Paper 2 style · calculator**

Let $f(x) = x^2 - 6x + 11$ for $1 \leq x \leq 7$.

Find the range of $f$. **[3]**

**3.** **3 marks · Paper 1 style · no calculator**

Let $f(x) = 4x + k$. Given that $(f \circ f)(2) = 37$, find $k$.

Find $k$. **[3]**

**4.** **3 marks · Paper 1 style · no calculator**

Let $f(x) = 5 - 2x^2$. Find $f(-3)$ and solve $f(x) = -27$.

**(a)** Find $f(-3)$. **[1]**

**(b)** Solve $f(x) = -27$. **[2]**

**5.** **3 marks · Paper 1 style · no calculator**

Let $h(x) = \sqrt{6 - 2x}$.

**(a)** Find the largest possible domain of $h$. **[2]**

**(b)** State the range of $h$ on this domain. **[1]**

**6.** **4 marks · Paper 1 style · no calculator**

Let $f(x) = \dfrac{3x + 1}{x - 2}$, $x \neq 2$.

**(a)** Find $f^{-1}(x)$. **[3]**

**(b)** State the domain of $f^{-1}$. **[1]**

**7.** 4 marks · Paper 1 style · no calculator

Let $f(x) = e^{2x}$ and $g(x) = \ln(x+1)$, $x > -1$.

**(a)** Find $(f \circ g)(x)$ in its simplest form. **[2]**

**(b)** Solve $(f \circ g)(x) = 9$. **[2]**

**8.** 5 marks · Paper 1 style · no calculator

Let $f(x) = 2x - 3$ and $g(x) = x^2 + 1$.

**(a)** Find $(g \circ f)(x)$. **[2]**

**(b)** Solve $(g \circ f)(x) = (f \circ g)(x)$. **[3]**

**9.** 6 marks · Paper 1 style · no calculator

Let $f(x) = \dfrac{x}{x+1}$, $x \neq -1$.

**(a)** Show that $(f \circ f)(x) = \dfrac{x}{2x+1}$. **[3]**

**(b)** Hence find $f^{-1}(x)$ and verify that $f^{-1}(f(x)) = x$. **[3]**

**10.** **7 marks · Paper 1 style · no calculator**

Let $f(x) = x^2 - 4x$ for $x \geq 2$.

**(a)** Find $f^{-1}(x)$, stating its domain. **[4]**

**(b)** Solve $f(x) = f^{-1}(x)$. **[3]**

## Answers and worked solutions

### Question 1 — 2 marks

**Answer:** $x \in \mathbb{R}, x \neq \pm 3$

$x^2 - 9 \neq 0$ **M1**

$x \in \mathbb{R}, x \neq \pm 3$ **A1**

### Question 2 — 3 marks

**Answer:** $2 \leq f(x) \leq 18$

$f(x) = (x-3)^2 + 2$, minimum $2$ $M1$**A1**

$f(7) = 18$, so $2 \leq f(x) \leq 18$ **A1**

### Question 3 — 3 marks

**Answer:** $1$

$f(2) = 8 + k$ **A1**

$4(8+k) + k = 37$ **M1**

$k = 1$ **A1**

### Question 4 — 3 marks

**Answer:** (a) $-13$; (b) $x = \pm 4$

**(a)**

$-13$ **A1**

**(b)**

$2x^2 = 32$ **M1**

$x = \pm 4$ **A1**

### Question 5 — 3 marks

**Answer:** (a) $x \leq 3$; (b) $h(x) \geq 0$

**(a)**

$6 - 2x \geq 0$ **M1**

$x \leq 3$ **A1**

**(b)**

$h(x) \geq 0$ **A1**

### Question 6 — 4 marks

**Answer:** (a) $f^{-1}(x) = \dfrac{2x+1}{x-3}$; (b) $x \neq 3$

**(a)**

$y(x-2) = 3x + 1$ **M1**

$x(y-3) = 2y + 1$ **A1**

$f^{-1}(x) = \dfrac{2x+1}{x-3}$ **A1**

**(b)**

$x \neq 3$ **A1**

### Question 7 — 4 marks

**Answer:** (a) $(x+1)^2$; (b) $2$

**(a)**

$e^{2\ln(x+1)}$ **M1**

$(x+1)^2$ **A1**

**(b)**

$x + 1 = 3$ $(x + 1 > 0)$ **M1**

$x = 2$ **A1**

### Question 8 — 5 marks

**Answer:** (a) $4x^2 - 12x + 10$; (b) $x = \dfrac{6 \pm \sqrt{14}}{2}$

**(a)**

$(2x-3)^2 + 1$ **M1**

$4x^2 - 12x + 10$ **A1**

**(b)**

$(f \circ g)(x) = 2x^2 - 1$ **A1**

$2x^2 - 12x + 11 = 0$ **M1**

$x = \dfrac{6 \pm \sqrt{14}}{2}$ **A1**

### Question 9 — 6 marks

**Answer:** (b) $f^{-1}(x) = \dfrac{x}{1-x}$

**(a)**

$\dfrac{\frac{x}{x+1}}{\frac{x}{x+1} + 1}$ **M1**

$= \dfrac{x}{x + x + 1}$ **A1**

$= \dfrac{x}{2x+1}$ **A1 AG**

**(b)**

$f^{-1}(x) = \dfrac{x}{1-x}$ **A2**

$\dfrac{\frac{x}{x+1}}{1 - \frac{x}{x+1}} = x$ **R1**

### Question 10 — 7 marks

**Answer:** (a) $f^{-1}(x) = 2 + \sqrt{x+4}$, $x \geq -4$; (b) $5$

**(a)**

$y = (x-2)^2 - 4$ **M1**

$x = 2 + \sqrt{y+4}$ (positive root as $x \geq 2$) **A1R1**

$f^{-1}(x) = 2 + \sqrt{x+4}$, $x \geq -4$ **A1**

**(b)**

solve $f(x) = x$ **M1**

$x^2 - 5x = 0$ **A1**

$x = 5$ **A1**

Every solution is also on ibmathrevision.com/hlaa/function-fundamentals-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.