Exponentials and Logarithms: exam practice

10 questions · 44 marks · worked solutions and markschemes

Name _____ Date _____ Score / 44

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/exponentials-and-logarithms-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Simplify fully $\dfrac{(8x^6)^{\frac{2}{3}}}{2x^{-1}}$.

Simplify. **[3]**

2. **3 marks · Paper 1 style · no calculator**

Solve $27^{x-1} = \left(\dfrac{1}{9}\right)^{2x}$.

Find x. **[3]**

3. **4 marks · Paper 1 style · no calculator**

Solve $4^x - 6 \times 2^x + 8 = 0$.

Find the values of x. **[4]**

4. **4 marks · Paper 2 style · calculator**

Solve the equation $3^{2x+1} = 5^{x+2}$, giving your answer correct to 3 significant figures.

Find x. **[4]**

5. **5 marks · Paper 1 style · no calculator**

Solve the equation $\log_2(x+3) + \log_2(x-1) = 5$.

Find x. **[5]**

6. **5 marks · Paper 1 style · no calculator**

Solve $\log_3 x = 4\log_x 3$, for $x > 0$, $x \neq 1$.

Find the values of x. **[5]**

7. **4 marks · Paper 1 style · no calculator**

Solve $\ln(x^2 - 16) = \ln(6x)$.

Find x. **[4]**

8. **5 marks · Paper 1 style · no calculator**

Given that $\log_a 3 = p$ and $\log_a 5 = q$, express each of the following in terms of p and q.

(a) $\log_a 45$ **[2]**

(b) $\log_a \dfrac{25}{\sqrt{3}}$ **[2]**

(c) $\log_3 5$ **[1]**

9. **5 marks · Paper 2 style · calculator**

The number of bacteria in a culture is $N = 500e^{0.32t}$, where t is in hours.

(a) Find the number of bacteria after 6 hours. **[2]**

(b) Find the time taken for the number to reach $20\,000$. **[3]**

10. **6 marks · Paper 1 style · no calculator**

Show that $\log_9(2x+1) = \frac{1}{2}\log_3(2x+1)$, and hence solve $\log_3(x-1) = \log_9(2x+1)$.

(a) Show the identity. **[2]**

(b) Solve the equation. **[4]**

Answers and worked solutions

Question 1 — 3 marks

Answer: $2x^5$

$(8x^6)^{\frac{2}{3}} = 4x^4$ **A1A1**

$2x^5$ **A1**

Question 2 — 3 marks

Answer: $\frac{3}{7}$

$3^{3x-3} = 3^{-4x}$ $M1$**A1**

$x = \frac{3}{7}$ **A1**

Question 3 — 4 marks

Answer: $x = 1$, $x = 2$

let $u = 2^x$: $u^2 - 6u + 8 = 0$ **M1**

$u = 2$ or $u = 4$ **A1**

$x = 1$, $x = 2$ **A1A1**

Question 4 — 4 marks

Answer: $x = 3.61$

$(2x+1)\ln 3 = (x+2)\ln 5$ **M1**

$x(2\ln 3 - \ln 5) = 2\ln 5 - \ln 3$ **A1**

$x = \frac{2\ln 5 - \ln 3}{2\ln 3 - \ln 5}$ **A1**

$x = 3.61$ **A1**

Question 5 — 5 marks

Answer: 5

$\log_2\left((x+3)(x-1)\right) = 5$ **M1**

$x^2 + 2x - 3 = 32$ **A1**

$(x+7)(x-5) = 0$ **M1**

$x = 5$ (reject $x = -7$) **A1R1**

Question 6 — 5 marks

Answer: $x = 9$, $x = \frac{1}{9}$

$\log_x 3 = \frac{1}{\log_3 x}$ **M1**

$(\log_3 x)^2 = 4$ **A1**

$\log_3 x = \pm 2$ **A1**

$x = 9$, $x = \frac{1}{9}$ **A1A1**

Question 7 — 4 marks

Answer: 8

$x^2 - 16 = 6x$ **M1**

$(x-8)(x+2) = 0$ **A1**

$x = 8$ **A1**

$x = -2$ rejected as $6x < 0$ **R1**

Question 8 — 5 marks

Answer: (a) $2p + q$; (b) $2q - \frac{p}{2}$; (c) $\frac{q}{p}$

(a)

$\log_a(9 \times 5) = 2\log_a 3 + \log_a 5$ **M1**

$2p + q$ **A1**

(b)

$2\log_a 5 - \frac{1}{2}\log_a 3$ **M1**

$2q - \frac{p}{2}$ **A1**

(c)

$\frac{q}{p}$ **A1**

Question 9 — 5 marks

Answer: (a) 3410; (b) 11.5 hours

(a)

$500e^{1.92}$ **M1**

3410 **A1**

(b)

$e^{0.32t} = 40$ **M1**

$t = \frac{\ln 40}{0.32}$ **A1**

11.5 hours **A1**

Question 10 — 6 marks

Answer: (b) 4

(a)

$\log_9 u = \frac{\log_3 u}{\log_3 9}$ **M1**

$= \frac{\log_3 u}{2}$ **A1 AG**

(b)

$2\log_3(x-1) = \log_3(2x+1)$ **M1**

$(x-1)^2 = 2x + 1$ **A1**

$x^2 - 4x = 0$ **A1**

$x = 4$ ($x = 0$ rejected) **A1**

Every solution is also on ibmathrevision.com/hlaa/exponentials-and-logarithms-exam-practice
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